Second Chance Club Exam 2 Worksheet

1. I failed to recognize that the Fundamental Theorem of Calculus applied and instead skipped right to the long way. Next time I will examine the problem longer before starting and realize it should not be such a difficult equation to solve.

1a. $\oint_C xe^{xyz}\,dx + ye^{xyz}\,dy + ze^{xyz}\,dz \quad r(t) = \langle t, t^2, t^3 \rangle \quad 0 \le t \le 1$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xe^{xyz} & ye^{xyz} & ze^{xyz} \end{vmatrix} = \left(\frac{\partial}{\partial y} ze^{xyz} - \frac{\partial}{\partial z} ye^{xyz}\right)i - \left(\frac{\partial}{\partial x} ze^{xyz} - \frac{\partial}{\partial z} xe^{xyz}\right)j + \left(\frac{\partial}{\partial x} ye^{xyz} - \frac{\partial}{\partial y} xe^{xyz}\right)k$$

$$= \left(\frac{z}{zx} e^{xyz} - \frac{y}{xy} e^{xyz}\right)i - \left(\frac{z}{yz} e^{xyz} - \frac{x}{xy} e^{xyz}\right)j + \left(\frac{y}{yz} e^{xyz} - \frac{x}{xz} e^{xyz}\right)k$$

$= 0i + 0j + 0k$ conservative

$f_x = xe^{xyz} \qquad f_y = ye^{xyz} \qquad f_z = ze^{xyz}$

$f = \int xe^{xyz}\,dx$

$u = x \qquad v = \frac{1}{yz} e^{xyz}$

$du = dx \qquad dv = e^{xyz}\,dx$

$f = \frac{x}{yz} e^{xyz} - \frac{1}{yz} \int e^{xyz}\,dx$

$f = \frac{x}{yz} e^{xyz} - \frac{1}{(yz)^2} e^{xyz} + g(y,z)$

$$f_y = \frac{yz \cdot (xe^{xyz} \cdot xz) - xe^{xyz} \cdot y}{(yz)^2} - \frac{(y^2z^2 \cdot e^{xyz} \cdot xz) - e^{xyz} \cdot (2y \cdot z^2 + 0 \cdot y^2)}{(y^2z^2)^2} + g_y(y,z)$$

$$f_y = \frac{xe^{xyz}(xyz^2 - y)}{y^2z^2} - \frac{e^{xyz}(xy^2z^3 - 2yz^2)}{y^4z^4} + g_y(y,z)$$

$$f_y = \frac{xe^{xyz}(xy^2z^2 - y^2z^2)}{y^4z^4} - \frac{e^{xyz}(xy^2z^3 - 2yz^2)}{y^4z^4} + g_y(y,z)$$

$$ye^{xyz} = \frac{xy^2z^2e^{xyz} - yze^{xyz} - xyze^{xyz} + 2e^{xyz}}{y^3z^2} + g_y(y,z)$$

$$\int g_y(y,z) = \int \frac{y^4z^2e^{xyz} - xy^2z^2e^{xyz} + yze^{xyz} + xyze^{xyz} - 2e^{xyz}}{y^3z^2}$$

$$g(y,z) = \frac{e^{xyz}(-zy^3x + yzx^2 - x^2 + y^2)}{x^2y^2z^2}$$

$$h(z) = \frac{e^{xyz}(x^3yz + zy^3x + yxz^3 - yz^2 - y^2 - z^2)}{y^2z^2x^2}$$

$$f = \frac{e^{xyz}(x^3yz + zy^3x + yxz^3 - yzx^2 - y^2 - z^2)}{y^2z^2x^2} - \frac{e^{xyz}(-zy^3x + yzx^2 - x^2 + y^2)}{x^2y^2z^2} + \frac{xe^{xyz}}{yz} - \frac{e^{xyz}}{y^2z^2}$$

$f(1,1,1) - f(0,0,0) = 0 - \text{DNE} = \text{DNE}$

I had to use maple for this problem.

1a cont. $x = t$ $y = t^2$ $z = t^3$ $dz = 1$ $dy = 2t$ $dz = 3t^2$

$$\int_0^1 te^{t^6} + 2t^3 e^{t^6} + 3t^5 e^{t^6}\, dt$$

I used maple to solve

Still ONE

I started out by thinking I would be able to use the Fundamental Theorem of Line Integral. However, when trying to find the function $F = \nabla f$, it got very complicated so I used Maple. It failed when I got ONE due to it being divided by 0. I tried a different method of find $dx$, $dy$, and $dz$ to fill in and solve. However this also gave ONE.

1b. $\int_C (4x^3y^2+1)\,dx + (2x^4y+1)\,dy$

$r(t)=\langle \sin t^2, \cos t^2\rangle \quad 0\le t\le \sqrt{\frac{\pi}{2}}$

$r(\sqrt{\tfrac{\pi}{2}})=\langle \sin\tfrac{\pi}{2}, \cos\tfrac{\pi}{2}\rangle = \langle 1,0\rangle$

$r(0)=\langle \sin 0, \cos 0\rangle = \langle 0,1\rangle$

$\frac{\partial}{\partial y}\, 4x^3y^2+1 = \frac{\partial}{\partial x}\, 2x^4y+1$

$8x^3y = 8x^3y$ ✓ conservative

$f_x = 4x^3y^2+1$

$f=\int 4x^3y^2+1\,dx$

$f=\cancel{4}y^2\frac{x^4}{\cancel{4}}+x+g(y)$

$f=y^2x^4+x+g(y)$

$f_y = 2yx^4+g_y(y)$

$2x^4y+1 = 2yx^4+g_y(y)$

$1=g_y(y)$

$y=g(y)$

$f=y^2x^4+x+y$

grad $=\langle 4x^3y^2+1,\ 2yx^4+1\rangle$ ✓

$f(1,0)-f(0,1)=$

$(0^2\cdot 1^4+1+0)-(1^2 0^4+1+0)=$

$1-1=0$

Since F is conservative, the equation f can be found to satisfy $\nabla F=f$. f is then used to evaluated f(end) - f(start). In this case f(end) = $f(r(\sqrt{\tfrac{\pi}{2}}))$ and f(start) = $f(r(0))$. The answer is 0.

2. I got this one correct.

2a. $\int_0^1\int_0^{e^x} f(x,y)\,dy\,dx$

$0\le x\le 1 \quad 0\le y\le e^x$

$\boxed{\begin{array}{l}0\le x\le \ln y\\ 0\le y\le e\end{array}}$

$\ln y\le \ln e^x$

$\ln y\le x$

$\int_0^e\int_0^{\ln y} f(x,y)\,dx\,dy$

2b. $\int_0^{\pi}\int_0^{\sin x} f(x,y)\,dy\,dx$ $\quad 0 \le x \le \pi \quad 0 \le y \le \sin x$

$\int_0^{1}\int_0^{\sin^{-1}y} f(x,y)\,dx\,dy$ $\quad \boxed{0 \le x \le \sin^{-1} y \quad 0 \le y \le 1}$

sinx max from 0 to $\pi$ is 1

2c. $\int_0^{1}\int_{e^y}^{e} f(x,y)\,dx\,dy$ $\quad e^y \le x \le e \quad 0 \le y \le 1$

$\ln e^y \le \ln x$

$y \le \ln x$

$\int_1^{e}\int_0^{\ln x} f(x,y)\,dy\,dx$ $\quad \boxed{\begin{matrix} 0 \le y \le \ln x \\ 1 \le x \le e \end{matrix}}$

3. I got this one correct

3a. Find tangent plane at point $(1,2,4)$ to surface
$x(u,v) = u^2, \quad y(u,v) = uv, \quad z(u,v) = v^2 \quad -\infty < u < \infty$ and $v$

$r = \langle u^2, uv, v^2 \rangle$

$r_u = \langle 2u, v, 0 \rangle \quad r_u(1,2) = \langle 2, 2, 0 \rangle$

$r_v = \langle 0, u, 2v \rangle \quad r_v(1,2) = \langle 0, 1, 4 \rangle$

$r_u \times r_v = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 0 & 1 & 4 \end{vmatrix} = (8-1)i - 8j + 2k$

$= 7i - 8j + 2k \quad \langle 7, -8, 2 \rangle$

$7(x-1) - 8(y-2) + 2(z-4) = 0$

$7x - 8y + 2z - 7 + 16 - 8 = 0$

$2z = -7x + 8y - 1$ $\quad \boxed{z = -\frac{7x}{2} + 4y - \frac{1}{2}}$

3b. Find tangent plane at point $(-1,-1,2)$
$x(u,v) = u^3 \quad y(u,v) = v^3 \quad z(u,v) = -2uv \quad -\infty < u$ and $v < \infty$

$r = \langle u^3, v^3, -2uv \rangle$

$r_u = \langle 3u^2, 0, -2v \rangle \quad r_u(-1,-1) = \langle 3, 0, 2 \rangle$

$r_v = \langle 0, 3v^2, -2u \rangle \quad r_v(-1,-1) = \langle 0, 3, 2 \rangle$

$$\begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 0 & 3 & 2 \end{vmatrix} = -6i - 6j + 9k \quad \langle -6, -6, 9 \rangle$$

$$-6(x+1) - 6(y+1) + 9(z-2) = 0$$

$$-6x - 6y + 9z - 30 = 0$$

$$9z = 6x + 6y + 30$$

$$\boxed{z = \tfrac{2}{3}x + \tfrac{2}{3}y + \tfrac{10}{3}}$$

4. I got this one correct

4a. $f(x,y,z) = \sin(x+y^2+z^3)$, $F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$, C is $r(t) = \langle t, t^2, t^3 \rangle$ $0 \le t \le 3$

Find $\int_C F \cdot dr$

$r(0) = \langle 0, 0, 0 \rangle$

$r(3) = \langle 3, 9, 27 \rangle$

$$F \times \nabla = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(x+y^2+z^3) & 2y\cos(x+y^2+z^3) & 3z^2\cos(x+y^2+z^3) \end{vmatrix} =$$

$= 0i + 0j + 0k$ conservative

$$f(3,9,27) - f(0,0,0) = \sin(3+9^2+27^3) - \sin(0)$$

$$= \sin(19767) = .0988$$

Since $F \times \nabla$ is 0, F is a conservative function and the Fundamental theorem of line Integrals can be used. Since f is already given all that needs to be done is $\int F \cdot dr = f(\text{end}) - f(\text{start})$ where $f(\text{end}) = f(r(3))$ and $f(\text{start}) = f(r(0))$

4b $f(x,y) = e^{\cos x + 3\sin y}$, $F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$ C is $r(t) = \langle \sin 2t, \cos t \rangle$ $0 \le t \le \pi$

$$F = \langle -\sin x e^{\cos x + 3\sin y}, 3\cos y\, e^{\cos x + 3\sin y} \rangle$$

$$\frac{\partial}{\partial y} -\sin x\, e^{\cos x + 3\sin y} = \frac{\partial}{\partial x} 3\cos y\, e^{\cos x + 3\sin y}$$

$$-\sin x \cdot 3\cos y\, e^{\cos x + 3\sin y} = 3\cos y \cdot -\sin x \cdot e^{\cos x + 3\sin y} \checkmark$$

conservative

$r(\pi) = \langle \sin 2\pi, \cos \pi \rangle = \langle 0, -1 \rangle$
$r(0) = \langle \sin 0, \cos 0 \rangle = \langle 0, 1 \rangle$

$f(0,-1) - f(0,1) = e^{\cos 0 + 3\sin(-1)} - e^{\cos 0 + 3\sin 1}$
$= \boxed{e^{1+3\sin -1} - e^{1+3\sin(1)}}$

Since F is conservative, the Fundamental Theorem of Line Integrals can be applied so $\int F \cdot dr =$ f(end) - f(start) with f(end) = f(r(π)) and f(start) = f(r(0))

5. I failed to recognize that spherical coordinates were needed to solve this problem. $dx\,dy\,dz = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$
$x^2 + y^2 + z^2 = \rho^2$

5a. Evaluate the triple integral $\int_R (x+y)(x^2+y^2+z^2)^2 \, dx\,dy\,dz$
$\{(x,y,z) \mid x^2+y^2+z^2 \le 1,\ x \ge 0,\ y < 0,\ z < 0\}$
$0 \le \rho \le 1 \qquad \pi \le \theta \le 2\pi \qquad \frac{\pi}{2} \le \phi \le \pi$

$\int_{\pi}^{2\pi}\int_{\pi/2}^{\pi}\int_{0}^{1} \rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta\,(\rho^2) \cdot \rho^2\sin\phi \, d\rho\, d\phi\, d\theta$

$\int_{\pi}^{2\pi}\int_{\pi/2}^{\pi}\int_{0}^{1} \rho^5 \sin^2\phi\,(\cos\theta + \sin\theta)\, d\rho\, d\phi\, d\theta$

$\int_{\pi}^{2\pi}\int_{\pi/2}^{\pi} \frac{\rho^6}{6}\Big|_0^1 \sin^2\phi\,(\cos\theta + \sin\theta)\, d\phi\, d\theta$

$\int_{\pi}^{2\pi} \frac{1}{6}\left(\sin^3\phi \cdot \frac{1}{3}\right|_{\pi/2}^{\pi} (\cos\theta + \sin\theta)\, d\theta$

$\frac{-1}{18}\int_0^{2\pi} \cos\theta + \sin\theta \, d\theta$

$\frac{-1}{18}(\sin\theta - \cos\theta)\Big|_{\pi}^{2\pi}$

$\frac{-1}{18}(0-1) - (0+1) = \frac{2}{18} = 9$

5b. Evaluate the Triple Integral $\int_R z(x^2+y^2+z^2)\,dx\,dy\,dz$

$R \quad \{(x,y,z) \mid x^2+y^2+z^2 \leq 1 \quad y \leq 0$

$0 \leq p \leq 1 \qquad \pi \leq \theta \leq 2\pi \qquad 0 \leq \phi \leq \pi$

$$\int_{\pi}^{2\pi}\int_0^{\pi}\int_0^1 p\cos\phi \; p^2 \cdot p^2 \sin\phi \; dp\,d\phi\,d\theta$$

$$\int_{\pi}^{2\pi}\int_0^{\pi}\int_0^1 p^5 \cos\phi \sin\phi \; dp\,d\phi\,d\theta$$

$$\int_{\pi}^{2\pi}\int_0^{\pi} \frac{p^6}{6}\Big|_0^1 \cos\phi\sin\phi \; d\phi\,d\theta \qquad \cancel{\frac{d}{d\theta}} \sin^2\theta = 2\sin\theta\cos\theta$$

$$\frac{1}{6}\int_{\pi}^{2\pi} \frac{1}{2}\sin^2\phi \; d\theta = \int_{\pi}^{2\pi} \frac{1}{12}(\sin^2\pi - \sin^2 0)\,d\theta$$

$$\int_{\pi}^{2\pi} 0 \; d\theta = 2\pi - \pi = \pi$$

5c. Evaluate the triple integral $\int_R (z-x)\,dx\,dy\,dz$

$R \; \{(x,y,z) \mid x^2+y^2+z^2 \leq 8\}$

$0 \leq p \leq \sqrt{8} \qquad 0 \leq \phi \leq \pi \qquad 0 \leq \theta \leq 2\pi$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{8}} (p\cos\phi - p\sin\phi\cos\theta) \cdot p^2\sin\phi \; dp\,d\phi\,d\theta$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{8}} p^3\cos\phi\sin\phi - p^3\sin^2\phi\cos\theta \; dp\,d\phi\,d\theta$$

$$\int_0^{2\pi}\int_0^{\pi} \frac{p^4}{4}\Big|_0^{\sqrt{8}} \cos\phi\sin\phi - \sin^2\phi\cos\theta \; d\phi\,d\theta$$

$$16\int_0^{2\pi} \frac{1}{2}\sin^2\phi - \sin^3\phi \cdot \frac{1}{3}\cos\theta \Big|_0^{\pi} d\theta$$

$$16\int_0^{2\pi} (0-0)-(0-0)\; d\theta$$

$$16(2\pi - 0) = 32\pi$$

6. I did not convert to polar coordinates. I should have seen $\sqrt{9-x^2}$ as a sign.

6a. convert to polar coordinates, do not evaluate $\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}}(x^2+y)\,dy\,dx$

$-3 \le x \le 0$ $\quad 0 \le y \le \sqrt{9-x^2}$

$0 \le y \le 3$

$0 \le r \le 3$

$\frac{\pi}{2} \le \theta \le \pi$

$$\int_{\pi/2}^{\pi}\int_{0}^{3}(r^2\cos^2\theta + r\sin\theta)\,r\,dr\,d\theta$$

6b. Convert to polar coordinates, do not evaluate

$\int_{0}^{4}\int_{-\sqrt{16-x^2}}^{0}(x^2+y)\,dy\,dx$ $\quad -\sqrt{16-x^2} \le y \le 0 \quad 0 \le x \le 4$

$-\sqrt{16} \le y \le 0$

$-4 \le y \le 0$

$-\sqrt{16-x^2} = y$

$16-x^2 = y^2$

$0 \le r \le 4$

$$\int_{3\pi/2}^{2\pi}\int_{0}^{4}(r^2\cos^2\theta + r\sin\theta)\,r\,dr\,d\theta$$

6c. Convert to polar coordinates, do not evaluate

$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}}\int_{x}^{\sqrt{1-x^2}}(x^3+y^2)\,dy\,dx$ $\quad x \le y \le \sqrt{1-x^2} \quad -\frac{\sqrt{2}}{2} \le x \le \frac{\sqrt{2}}{2}$

$y = \sqrt{1-x^2}$ $\quad \sqrt{1-\frac{2}{4}}$

$y^2 + x^2 = 1$

$$\int_{0}^{2\pi}\int_{0}^{1}(r^3\cos^3\theta + r^2\sin^2\theta)\,r\,dr\,d\theta \quad r=1$$

7. for d I needed to approach the limit from 2 different directions, meaning limit parallel to y axis, x axis, and z axis

Decide whether the following limits exist. If it does, find it, if not, explain!

7a. $\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3} = \frac{0}{0}$ $\qquad y-3=c(x-1)$

$y=cx-c+3$

$\lim_{(x,y)\to(1,3)} \frac{x-1}{cx-c+3-3} = \frac{x-1}{cx-c}$

DNE because it depends on the slope

7b. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{0}{0}$

$\lim_{y\to 0} \frac{0+y+0}{0+y+0} = \frac{y}{y} = 1$

$\lim_{x\to 0} \frac{x+0+0}{2x+0+0} = \frac{x}{2x} = \frac{1}{2}$

Since the limit in the y axis plane is different than the x axis, it DNE

8. I got this one correct

8a. Compute the line integrals $\int_c f\,ds$ $\quad f(x,y,z)=xy^2+yz^2+z$
(0,0,0) to (1,1,-1)

$\langle 0,0,0\rangle + t\langle 1,1,-1\rangle = \langle t,t,-t\rangle$

$r'(t)=\langle 1,1,-1\rangle \qquad |r'(t)| = \sqrt{1+1+1} = \sqrt{3}$

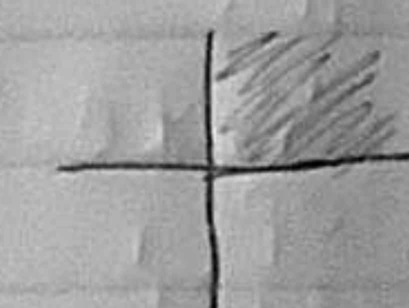
$\int_c f\,ds = \int_0^1 t^3+t^3-t\sqrt{3}\,dt = \sqrt{3}\,\frac{2t^4}{4} - \frac{t^2}{2}\Big|_0^1 = \sqrt{3}(0) = 0$

9b. Compute the vector field surface integral $\iint_S F \cdot dS$ if $F = \langle x+z, y+z, -x \rangle$ S is $z = 9 - x^2 - y^2$ $0 \le x \le 1$ $0 \le y \le 1$ $z \ge 0$ downward

$g(x,y) = 9 - x^2 - y^2$

$z = 9 - 0 - 0$

$z = 9$



$P = x + z$ $\quad$ $Q = y + z$ $\quad$ $R = -x$

$\frac{dg}{dx} = -2x$ $\quad$ $\frac{dg}{dy} = -2y$

$$\iint_R 2x(x+z) + 2y(y+z) - x \, dA$$

$$\int_0^1 \int_0^1 2x^2z + xz^2 + 2y^2z + yz^2 - xz \Big|_0^9 \, dy\, dz$$

$$\int_0^1 \int_0^1 18x^2 + 81x + 18y^2 + 81y - 9x \, dx\, dy = \frac{18x^3}{3} + \frac{81x^2}{2} + 18y^2x + 81yx - \frac{9x^2}{2} \Big|_0^1$$

$$\int_0^1 42 + 18y^2 + 81y \, dy = \frac{\cancel{42}^{84}}{2} + \frac{\cancel{6}^{12}}{2} + \frac{81}{2} = \frac{-167}{2}$$

10. I tried to solve the problem without using the equation of the plane given. Therefore, I got stuck and could not find an answer.

10a. Find the maximum value of the function $f(x,y,z) = xyz$ on the plane $2x + y + z = 4$

$\nabla f = \langle yz, xz, xy \rangle$ $\quad$ $\nabla g = \langle 2, 1, 1 \rangle$

$\langle yz, xz, xy \rangle = \lambda \langle 2, 1, 1 \rangle$

$yz = 2\lambda$ $\quad$ $xz = \lambda$ $\quad$ $xy = \lambda$

$2x + y + z = 4$

$y + y + y = 4$

$3y = 4$

$y = \frac{4}{3}$

$\frac{y\cancel{z}}{x\cancel{z}} = \frac{2\cancel{\lambda}}{\cancel{\lambda}}$ $\quad$ $y = 2x$

$\frac{4}{3} = 2x$

$\frac{1}{3} = x$

$\frac{\cancel{x}z}{\cancel{x}y} = \frac{\cancel{\lambda}}{\cancel{\lambda}}$ $\quad$ $z = y$

$z = \frac{4}{3}$

$f(\frac{1}{3}, \frac{4}{3}, \frac{4}{3}) = (\frac{1}{3})(\frac{4}{3})(\frac{4}{3}) = \frac{16}{27}$

10b. Find the point on the plane $2x+y+z=4$ where $f(x,y,z)=xy^2z$ is as large as possible

$\nabla f = \langle y^2z, 2yxz, xy^2 \rangle$ $\nabla g = \langle 2,1,1 \rangle$

$y^2z = 2\lambda$ $2yxz = \lambda$ $xy^2 = \lambda$

$\frac{\cancel{y^2}z}{x\cancel{y^2}} = \frac{2\cancel{\lambda}}{\cancel{\lambda}}$ $2\cancel{y}\cancel{x}z = \cancel{x}y^{\cancel{2}}$

$z = 2x$ $2z = y$

$2x+y+z=4$ $z=2x$ $2z=y$

$z+2z+z=4$ $1=2x$ $2(1)=y$

$4z=4$ $x=\frac{1}{2}$ $2=y$

$z=1$

$\left(\frac{1}{2}, 2, 1\right)$