Fayed Raza 12/13/20

MATH 251 (22,23,24), Dr. Z. Second Chance Club for Exam 2 Worksheet

Due Friday, Dec. 13, 2020, 8:00pm. Email an attachment called scc2FirstLast.pdf to DrZcalc3@gmail.com Subject: scc2

Make sure that you have the posted solutions:

`http://www.math.rutgers.edu/~zeilberg/calc3NNN/mt2S.pdf`

and understand **each** problem.

$F = xyz + yz + h(z)$

$F_z = \cancel{xy} + \cancel{y} + h'(z) = \cancel{xy} + \cancel{y} + 1$

$h(z) = z$

In this worksheet, you are supposed to state what was your error for each of the exam questions (if you made an error) and say how to avoid it in the future. Then regardless of whether you got it right or wrong do the similar problems given.

$xyz + yz + z$

$f_x = yz$

$F = xyz + h(y,z)$

$\cancel{xz} + h'(y) = \cancel{xz} + z$

$h'(y) = z$ $h(y) = yz$

Exam Problem 1. (10 pts.) Compute the line integral

$$\int_C yz\,dx + (xz+z)\,dy + (xy+y+1)\,dz \quad ,$$

$\langle yz, xz+z, xy+y+1 \rangle$

$r(0) = \langle 1,0,0 \rangle$

over the path

$$\mathbf{r}(t) = \langle e^{t^3}, t^2e^{t^4}, te^{t^7} \rangle \quad , \quad 0 \le t \le 1 \quad .$$

$r(1) = \langle e, e, e \rangle$

Explain!

First find function F and then find r(1) and r(0) because $0 \le t \le 1$ then $F(r(1)) - F(r(0))$.

$eee + ee + e - 0 - 0 + 1$

$e^3 + e^2 + e$

:

Here is what I did wrong (if applicable): I didnt find function F which I should have find first

Problem 1a. Compute the line integral

$$\int_C xe^{xyz}\,dx + ye^{xyz}\,dy + ze^{xyz}\,dz \quad ,$$

over the path

$F = e^{xyz}$ $r(1) = \langle 1,1,1 \rangle$

$$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \quad , \quad 0 \le t \le 1 \quad .$$

$r(0) = \langle 0,0,0 \rangle$

Explain!

$e^1 - e^0 = e^1 - 1$

We determine it to be $F = e^{xyz}$ and $r(1) = \langle 1,1,1 \rangle$ and $r(0) = \langle 0,0,0 \rangle$. We find the difference between them.

Problem 1b. Compute the line integral

$$\int_C (4x^3y^2+1)\,dx + (2x^4y+1)\,dy \quad ,$$

over the path

$$\mathbf{r}(t) = \langle \sin t^2, \cos t^2 \rangle \quad , \quad 0 \le t \le \sqrt{\pi/2} \quad .$$

Explain!

: It can't be determined due to dz does not exist.

Exam Problem 2. (10 points) By changing the order of integration, if necessary, evaluate the double-integral

$$\int_0^5 \int_{(y/5)^{1/3}}^1 \sin x^4 \, dx \, dy$$

$y=0$ $x=1$

$y=5$ $x=(y/5)^{1/3}$

$5x^3=y$

$y=1$ $y=0$ $x=1$

$$\int_0^1 \int_0^{5x^3} (\sin(x))^4 \, dy \, dx$$

$\int_0^1 5x^3 \sin(x^4)$

$4x^3dx$

$\frac{5}{4}\int_0^1 \sin(u)\,du$

$-\cos(1)+\cos(0)$

$\frac{5}{4}(-\cos(1)+1)$

Here is what I did wrong (if applicable):
I misunderstood $\sin x^4$ as $\sin(x^4)$

Problem 2a:

Change the order of integration

$y=e^x$ $x=1$

$y=0$ $x=0$

$$\int_0^1 \int_0^{e^x} f(x,y) dy \, dx$$

$y=0$ $x=0$ $x=1$

$$\int_1^e \int_0^{\ln(x)} f(x,y)\,dx\,dy$$

2

Problem 2b:

Change the order of integration

$y=\sin x$ $y=0$

$$\int_0^\pi \int_0^{\sin x} f(x,y) dy \, dx$$

$y=0$ $x=0$ $x=\pi$

$$\int_0^1 \int_0^{\sin^{-1}(y)} f(x,y)\,dx\,dy$$

Problem 2c:

Change the order of integration

$$\int_0^1 \int_{e^y}^e f(x,y) dx \, dy$$

$y=1$ $y=0$ $x=e$

$$\int_0^e \int_0^{\ln(x)} f(x,y)\,dy\,dx$$

Exam Problem 3. (10 points) Find the equation of the tangent plane at the point $(1,1,1)$ to the surface given parametrically by

$$x(u,v) = u^3 v \quad , \quad y(u,x) = uv \quad , \quad z(u,v) = uv^3 \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

$\langle u^3v, uv, uv^3 \rangle$

$$\begin{vmatrix} i & j & k \\ 3u^2v & v & v^3 \\ u^3 & u & 3v^2u \end{vmatrix} \qquad \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$\langle 2, -8, 2 \rangle \cdot \langle x-1, y-1, z-1 \rangle$

$2(x-1) - 8(y-1) + 2(z-1)$

$2x - 2 - 8y + 8 + 2z - 2$

$2x - 8y + 2z + 4 = 0$

$2x - 8y + 4 = -2z$

$-x + 4y - 2 = z$

2

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

Here is what I did wrong (if applicable):

I misunderstood what to do

Problem 3a. Find the equation of the tangent plane at the point $(1, 2, 4)$ to the surface given parametrically by

$$x(u, v) = u^2 \quad , \quad y(u, x) = uv \quad , \quad z(u, v) = v^2 \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

$\begin{vmatrix} 2u & v & 0 \\ 0 & u & 2v \end{vmatrix}$

$\begin{vmatrix} 2 & 2 & 0 \\ 0 & 1 & 4 \end{vmatrix}$

$8i - 8j + 2k$

$8(x-1) - 8(y-2) + 2(z-4)$

$8x - 8y + 16 - 2z$

$8x - 8 - 8y + 16 + 2z - 8$

$z = -8 + 4y - 4x$

Problem 3b. Find the equation of the tangent plane at the point $(-1, -1, 2)$ to the surface given parametrically by

$$x(u, v) = u^3 \quad , \quad y(u, x) = v^3 \quad , \quad z(u, v) = -2uv \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

$\begin{vmatrix} 3u^2 & 0 & -2v \\ 0 & 3v^2 & -2u \end{vmatrix}$

$\begin{vmatrix} 3 & 0 & 2 \\ 0 & 3 & 2 \end{vmatrix}$

$-6(x+1) + 6(y+1) + 9(z-2)$ $\langle 6, 6, 9 \rangle$

$-6x - 6 + 6y + 6 + 9z - 18$

$-6x + 6y - 18 = -9z$

$\frac{2}{3}x - \frac{2y}{3} + 2 = z$

Exam Problem 4. Let $f(x, y, z) = e^{\cos x^2 + \sin xyz + \cos xz}$, and let

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

Let C be the curve

$$r(t) = \langle \cos t, t, \sin t \rangle \quad , \quad 0 \le t \le 2\pi \quad .$$

Find the value of the line-integral

$$\int_C \mathbf{F} . d\mathbf{r} \quad .$$

$f(1,0)$

$r(0) = \langle 1, 0, 0 \rangle$

$r(\pi) = \langle -1, 2\pi, 0 \rangle$

Explain! Just giving the answer will give you no credit.

We use the Fundemental theorem of line integral

$f(-1, 2\pi, 0) - f(1, 0, 0)$

$e^{\cos 1 + 1} - e^{\cos 1 + 1} = 0$

Here is what I did wrong (if applicable):

I didn't use the fundemental theorem

Problem 4a Let $f(x, y, z) = \sin(x + y^2 + z^3)$, and let

$f(1,0)$

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

$\sin(3 + 81 + 19683$

$\sin(3 + 9^2 + 27^3) - \sin(0)$ $\sin(19767)$

Let C be the curve

$$r(t) = \langle t, t^2, t^3 \rangle \quad , \quad 0 \le t \le 3 \quad .$$

$\langle 0, 0, 0 \rangle$

$\langle 3, 9, 27 \rangle$

Find the value of the line-integral

$$\int_C \mathbf{F}.d\mathbf{r} \quad .$$

Explain! Just giving the answer will give you no credit.

I use the fundamental line theorem.

Problem 4b Let $f(x, y) = e^{cos x + 3 \sin y}$, and let

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$$

$r(0) = \langle 0, 1 \rangle$

Let C be the curve

$$r(t) = \langle \sin 2t, \cos t \rangle \quad , \quad 0 \le t \le \pi \quad .$$

$r(\pi) = \langle 0, -1 \rangle$

Find the value of the line-integral

$$\int_C \mathbf{F}.d\mathbf{r} \quad .$$

$e^{\cos(0) + 3\sin(-1)} - e^{\cos(0) + 3\sin(1)}$

Explain! Just giving the answer will give you no credit.

I use the fundamental line theorem.

$e^{1 + 3\sin(-1)} - e^{1 + 3\sin(1)}$

Exam Problem 5. (10 points) Evaluate the triple integral

$$\int_R (x^2 + y^2 + z^2)^3 \, dx \, dy \, dz$$

$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 \rho^8 \sin\phi \, d\rho \, d\phi \, d\theta$

where R is the region in 3D space given by

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \le 1 \quad , \quad x, y, z \ge 0\} \quad .$$

$\frac{\rho^9}{9} \Big|_0^1$

$\frac{1}{9} -\cos\phi \Big|_0^{\frac{\pi}{2}}$

$\int_0^{\frac{\pi}{2}} \frac{1}{9} \sin\phi$

$-\cos(\frac{\pi}{2}) + \cos(0)$

$0 + (1) = 1$

Here is what I did wrong (if applicable):

I determined the bounds wrong since $x, y, z \ge 0$

$\int_0^{\frac{\pi}{2}} \frac{1}{9} \, d\theta$

$\frac{\pi}{18}$

Problem 5a Evaluate the triple integral

$$\int_R (x + y)(x^2 + y^2 + z^2)^2 \, dx \, dy \, dz \quad ,$$

where R is the region in 3D space given by

$\int\int\int (\rho^6)(\rho^2) \sin\phi \, r$

$\int_0^{\pi}\int_0^{\pi}\int_0^1 \rho^8 \sin\phi \, r(\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta) \, d\rho \, d\phi \, d\theta$

$\frac{\rho^9}{9} \sin\phi\cos\theta \; \frac{\rho^2}{2} \sin\phi\sin\theta$

$\frac{1}{9}\sin^2\phi\cos\theta + \frac{1}{2}\sin\phi\sin\theta$

$\frac{\sin 3\phi}{3}$

$\frac{1}{27}\cos 3\theta + \frac{1}{27}\sin 3\theta\cos\theta$

0

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \le 1 \quad , \quad x \ge 0, y < 0, z < 0\} \quad .$$

Problem 5b Evaluate the triple integral

$$\int_R z(x^2 + y^2 + z^2)\, dx\, dy\, dz \quad ,$$

$$\int_0^{\frac{3\pi}{2}} \int_0^{\frac{3\pi}{2}} \int_0^1 \rho^5 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$$

where R is the region in 3D space given by

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \le 1 \quad , \quad y < 0\} \quad .$$

$$\int_0^1 \rho^3 \, d\rho$$

$$\int_0^{\frac{3\pi}{2}} \frac{1}{12} \sin(2\phi)$$

$$0$$

Problem 5c Evaluate the triple integral

$$\int_R (z - x)\, dx\, dy\, dz \quad ,$$

where R is the region in 3D space given by

$$\int_0^{2\pi} \int_0^{2\pi} \int_0^{\sqrt{8}} \rho(\sin\phi \cos\theta - \cos\phi)\, d\rho\, d\theta\, d\phi$$

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \le 8\} \quad .$$

$$\frac{\rho^2}{2} \quad 4 \quad -\cos\phi \cos\theta - \sin\phi \Big|_0^{2\pi}$$

$$-\cos\theta$$

$$-\sin\phi \Big|_0^{2\pi}$$

$$0$$

Exam Problem 6. Evaluate the double integral

$$\int_{-3}^{0} \int_{0}^{\sqrt{9-x^2}} (x^2 + y^2)^2 \, dy\, dx$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^3 r^5 \, dr\, d\theta$$

$$\frac{r^6}{6} \Big|_0^3 \qquad \frac{3^6}{6}$$

$$\frac{729}{12}\pi \qquad \frac{243}{4}\pi \qquad \int_{\frac{\pi}{4}}^{\pi} \frac{729}{6} d\theta$$

Here is what I did wrong (if applicable):

I set up my y bounds wrong 243π

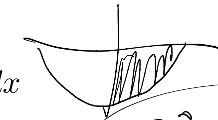
Problem 6a Convert the integral to polar coordinates, do not evaluate.

$$\int_{-3}^{0} \int_{0}^{\sqrt{9-x^2}} (x^2 + y)\, dy\, dx$$

$$\int_{\pi}^{\frac{\pi}{2}} \int_0^3 r^3 \cos^2\theta + r^2 \sin\theta \, dr\, d\theta$$

Problem 6b Convert the integral to polar coordinates, do not evaluate.

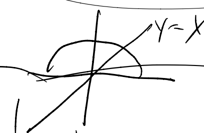
$$\int_0^4 \int_{-\sqrt{16-x^2}}^0 (x^2 + y)\, dy\, dx$$



$$\int_{-\frac{\pi}{2}}^{0} \int_0^4 r^3(\cos\theta + r\sin\theta)\, dr\, d\theta$$

Problem 6c Convert the integral to polar coordinates, do not evaluate.

$$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_x^{\sqrt{1-x^2}} (x^3 + y^2)\, dy\, dx$$



$y=x$

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{6}} \int_0^1 r^4(\cos^3\theta + r^3\sin^2\theta)\, dr\, d\theta$$

Exam Problem 7. (10 points altogether) Decide whether the following limits exist. If it does find them. If it does not **Explain** why not?

$$\lim_{r\to 0} \frac{r^2(\cos^2\theta - \sin^2\theta)}{r(\cos\theta - \sin\theta)} \quad 0$$

(*a*) (2 *points*) $\displaystyle\lim_{(x,y)\to(\pi/2,\pi/2)} \frac{\cos x + \sin x}{x+y}$ $\frac{1}{\pi}$, (*b*) (2 *points*) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$,

(*c*) (2 *points*) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2-y^2}$, (*d*) (4 *points*) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x+y-2}{2x+y-3}$, does not exist

$$\lim_{r\to 0} \frac{r(\cos\theta - \sin\theta)}{r^2(\cos^2\theta - \sin^2\theta)} = \lim \frac{1}{\cos\theta + \sin\theta} = \frac{1}{0}\ \infty \text{ dne}$$

$$\lim_{(1,1.0001)} \frac{1+1.0001-2}{2+1.0001-3} = 9.9\times 10^{-3}$$

Here is what I did wrong (if applicable):

$$\lim_{(1.0001,1)} \frac{1.0001+1-2}{2(1.0001)+1-3} = .3 \quad \neq$$

Problem 7a: Decide whether the following limit exists. If it does, find it, if not, explain!

$$\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3},$$

The left and right bounds do not equal to each other so it does not exist.

$$\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3} = \frac{0}{0}$$

$$\lim_{(x,y)\to(1,3.001)} = \frac{1-1}{3.001-3} = 0$$

$$\lim_{(1.001,3)} \frac{1.001-1}{3-3} = \text{undefined}$$

Problem 7b: Decide whether the following limit exists. If it does, find it, if not, explain!

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z},$$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{0}{0}$$

$$\lim_{(1+t,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{1+t+1+z}{2(1+t)+1+1}$$

$$= \frac{\frac{1}{t}+\frac{4}{t}}{2t+\frac{4}{t}} = \frac{1}{2}$$

Exam Problem 8. Compute the line integral $\int_C f\, ds$ where

$$f(x,y,z) = xyz$$

and C is the line segment starting at $(0,0,0)$ and ending at $(1,2,-3)$

$1, 2, -3$

$r(t) = \langle t, 2t, -3t \rangle$ $\quad r'(t) = \langle 1, 2, -3 \rangle$ $\quad |r'(t)| = \sqrt{14}$

$$\int_0^1 -6t^3\sqrt{14}\, dt \quad \frac{t^4}{4} \quad -\frac{6\sqrt{14}}{4} \quad -\frac{3\sqrt{14}}{2}$$

Here is what I did wrong (if applicable):

I did not understand the theorem

Problem 8a Compute the line integral $\int_C f\, ds$ where

$r(t) = \langle t, t, -t \rangle$

$r'(t) = \langle 1, 1, -1 \rangle$

$$f(x, y, z) = xy^2 + yz^2 + z$$

and C is the line segment starting at $(0, 0, 0)$ and ending at $(1, 1, -1)$

$\int_0^1 (t^3 + t^3 + t)\sqrt{3}\, dt$ $\sqrt{3}$

$\frac{t^4}{4} + \frac{t^4}{4} + \frac{t^2}{2}\Big|_0^1$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{2}$ $\frac{2}{4} + \frac{2}{4} = 1$

Problem 8b Compute the line integral $\int_C f\, ds$ where

$$f(x, y) = x + y$$

$\langle 2x, 2y \rangle$

and C is the upper circle $\{(x, y) : x^2 + y^2 = 1, y > 0\}$.

$\langle 2, 2 \rangle$

2

$\int_0^1 \int_0^{2x+2y} \sqrt{8}\, dx\, dy$ $x^2 + 2xy$ $\int_0^1 1 + 2x\, dx$ $y + 1$

Exam Problem 9. Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle \overset{p}{z}, \overset{q}{z}, \overset{R}{x} \rangle \quad ,$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x \geq 0, y \geq 0, z \geq 0$$

with **downward pointing** normal.

$\int_0^3 \int_0^{\sqrt{9-x^2}} -(z)(-2x) - z(-2y) + x)\, dy\, dx$

$\frac{693}{5} \times -1 = -\frac{693}{5}$

Here is what I did wrong (if applicable):

I didn't use the formula

Problem 9a Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle \overset{p}{x + z}, \overset{q}{y + z}, \overset{r}{-x} \rangle \quad ,$$

$\int_0^3 \int_0^{\sqrt{9-x^2}} (-x - z(-2x)) - y - z(-2y) - x)\, dy\, dx$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x < 0, y < 0, z \geq 0$$

-121

with **upward pointing** normal.

Problem 9b Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle \overset{P}{x + z}, \overset{Q}{y + z}, \overset{R}{-x} \rangle \quad ,$$

$\int_0^3 \int_0^{\sqrt{9-x^2}} -(x+z)(-2x) - (y+z)(-2y) - x\,)dy\, dx$ $-\frac{257}{5}$

$\frac{257}{5} \times -1$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad ,0 < x < 1, 0 < y < 1, z \geq 0$$

with **downward pointing** normal.

Exam Problem 10. Find the **point** on the plane $x + 2y + 3z = 18$ where the function $f(x, y, z) = xyz$ is **as large as possible**.

$(6, 3, 2)$

$\langle 1, 2, 3 \rangle$ $\quad 1 = L(yz)$ $\quad 2 = L(xz)$ $\quad 3 = L(xy)$ $\quad \frac{1}{2} = \frac{yz}{xz}$ $\quad xz = 2yz$ $\quad \frac{x}{y} = 2$

$L\langle yz, xz, xy \rangle$

$3x = 18$, $x = 6$, $y = 3$, $z = 2$

Here is what I did wrong (if applicable):

I forgot to divide to find answer.

Problem 10a Find the **maximum value** of the function $f(x, y, z) = xyz$ on the plane $2x + y + z = 4$

$2x + 2x + 2x = 4$ $\quad 6x = 4$ $\quad x = \frac{4}{6}$ $\quad x = \frac{2}{3}$

$\frac{32}{27}$

$\langle 2, 1, 1 \rangle$ $\quad L\langle yz, xz, xy \rangle$

$2 = Lyz$ $\quad 1 = Lxz$ $\quad 2x = y$

Problem 10b Find the point on the plane $2x + y + z = 4$ where $f(x, y, z) = xy^2z$ is as large as possible. (You can use Maple)

$\langle 2, 4, 1 \rangle$

$\langle 2, 1, 1 \rangle$

$L\langle y^2z, 2yz, xy^2 \rangle$

$2 = Ly^2z$ $\quad 1 = L2yz$ $\quad 1 = Lxy^2$

$2 = Ly$ $\quad 1 = Lz$ $\quad 4 = y$

$x = 2$ $\quad y = 4$ $\quad z = 1$