Marathon Attendance Quiz - Shaun Goda

SCC 2

1 st attendance Question

find an equation for the plane passing through the points $(0,0,0), (1,2,3), (2,3,1)$

$\vec{PQ} = \langle 1, 2, 3 \rangle$ $PR = \langle 2, 3, 1 \rangle$

$$\vec{n} = PQ \times PR = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= (2-9)i - (1-6)j + (3-4)k$$

$$= \langle -7, 5, -1 \rangle$$

$$\boxed{-7x + 5y - z = 0}$$

2nd Attendance Question

find normal vector to $\langle 1,5,1 \rangle, \langle 4,-7,2 \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 5 & 1 \\ 4 & -7 & 2 \end{vmatrix} = (10+7)i - (2-4)j + (-7-20)k$$

$$\boxed{= \langle 17, 2, -27 \rangle}$$

3rd Attendance Problem

Find the curvature of the curve

$r(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$ @ $(1, 1, \frac{2}{3})$

$r'(t) = \langle 1, 2t, 2t^2 \rangle$

$r''(t) = \langle 0, 2, 4t \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 2t & 2t^2 \\ 0 & 2 & 4t \end{vmatrix} = (8t^2 - 4t^2)i - (4t)j + (2)k$$

$$= \langle 4t^2, -4t, 2 \rangle$$

$\dfrac{\sqrt{16t^4 + 16t^2 + 4}}{(1 + 4t^2 + 4t^4)^{\frac{3}{2}}}$ @ $t = 1$ equal $\boxed{\frac{2}{9}}$

4th Attendance Problem

Use implicit differentiation to find $\frac{\partial x}{\partial z}$ and $\frac{\partial z}{\partial y}$ given the relationship

$$x^3 + y^3 + z^4 = 3xy^2z$$

$$3x^2 \frac{\partial x}{\partial z} + 4z^3 = 3xy^2 + 3y^2z \frac{\partial x}{\partial z}$$

$$\frac{\partial x}{\partial z}(3x^2 - 3y^2z) = 3xy^2 - 4z^3$$

$$\boxed{\frac{\partial x}{\partial z} = \frac{3xy^2 - 4z^3}{3x^2 - 3y^2z}}$$

$$3y^2 + 4z^3 \frac{\partial z}{\partial y} = 6xyz + 3xy^2 \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial y}(4z^3 - 3xy^2) = 6xyz - 3y^2$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{6xyz - 3y^2}{4z^3 - 3xy^2}}$$

5th Attendance Problem

Find the linearalization of the function $f(x,y) = x^3y^3$ at the point $(1,1)$

$f_x = 3x^2y^3$ $\quad f_x(1,1) = 3$ $\quad f(1,1) = 1$

$f_y = 3x^3y^2$ $\quad f_y(1,1) = 3$

$L(x,y) = 1 + (3)(x-1) + (3)(y-1)$

$= 1 + 3x - 3 + 3y - 3$

$\boxed{3x + 3y - 5}$

6th Attendance Problem

let $f(x,y,z) = -x^2 + y^2 + z^2 - 1$

(a) compute ∇f

$\boxed{\nabla f = \langle -2x, 2y, 2z \rangle}$

(b) $(1,1,1) \cdot \langle -2x, 2y, 2z \rangle \Rightarrow \langle -2, 2, 2 \rangle$

$-2(x-1) + 2(y-1) + 2(z-1) = 0$

$-2x + 2 + 2y - 2 + 2z - 2 = 0$

$\boxed{-2x + 2y + 2z = 2}$

(c) $\nabla f = \langle -2x, 2y, 2z \rangle$

$\nabla f(1,1,1) = \langle -2, 2, 2 \rangle$

$\|v\| = \sqrt{9} = 3$ $\quad u = \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$D_u f(1,1,1) = \langle -2, 2, 2 \rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$= -\frac{2}{3} + \frac{4}{3} + \frac{4}{3}$

$\boxed{= 2}$

17th Attendance Problem $f(x,y,z)=$

Find the largest value that $x+3y+5z$ can, given $x^2+y^2+z^2=35$

$\nabla f = \langle 1, 3, 5 \rangle$
$\nabla g = \langle 2x, 2y, 2z \rangle$
$\nabla f = \lambda \nabla g \Rightarrow \langle 1, 3, 5 \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$1 = 2\lambda x \quad \times \quad 3 = 2\lambda y \quad \times \quad 5 = 2\lambda z$
$15 = 8\lambda^3 xyz$

$15 = 8\lambda^3 \left(\frac{1}{2\lambda}\right)\left(\frac{3}{2\lambda}\right) z \qquad 15 = 8\lambda^3 x \left(\frac{3}{2\lambda}\right)\left(\frac{5}{2\lambda}\right)$
$15 = 6\lambda z \qquad 15 = 30\lambda x$
$z = \frac{15}{6\lambda} = \frac{5}{2}\lambda \qquad x = \frac{1}{2}\lambda$

$15 = 8\lambda^3 \left(\frac{1}{2\lambda}\right) y \left(\frac{5}{2\lambda}\right)$
$15 = 10\lambda y$
$y = \frac{3}{2}\lambda$

$\left(\frac{1}{2}\lambda\right)^2 + \left(\frac{3}{2}\lambda\right)^2 + \left(\frac{5}{2}\lambda\right)^2 = 35$
$\frac{35}{4}\lambda^2 = 35 \qquad \lambda^2 = 4$
$\lambda = \pm 2$

@ $\lambda = 2$ $\qquad$ @ $\lambda = -2$
$(1, 3, 5) \qquad (-1, -3, -5)$

$f(1,3,5) = 35 \qquad f(-1,-3,-5) = -35$

$\boxed{\text{Maximum: } 35}$

$\boxed{\text{minimum: } -35}$

8th attendance Problem

Convert $\iint_R (36 - x^2 - y^2)\, dV$ for

$R \Rightarrow \{(x, y) : x^2 + y^2 < 36\}$ to polar coodinate

$$\iint_R (36 - x^2 - y^2)\, dv = -1 \iint_R (-36 + x^2 + y^2)\, dv$$

polar $\Rightarrow$ $-1 \int_0^6 \int_0^{2\pi} (-36 + r^2)\, r \, d\theta \, dr$

$$= \boxed{\int_0^6 \int_0^{2\pi} (36r - r^3)\, d\theta \, dr}$$

9th attandance Problem

let $F = \langle e^{x^3 + yz + \sin(x, y, +z)}, \sin(x^3 + yz + z^3), e^{\sin(x + y + z^3)} \rangle$,

and let $G = \text{curl}(F)$ where $G = \langle P, Q, R \rangle$

find $P_x + Q_y + R_z$

$\langle e^{x^3} \cos(x + y, z) e^{x^3 + y}, \sin(x^3 + yz + z^3), \sin(x^3 + yz + z)$