Second Chance Club Exam 2 Worksheet

Exam Problem 1. Did not realize to check for fundamental theorem of surface integrals and did it more complicated

1a. $r(0) = \langle 0,0,0\rangle$ $r(1) = \langle 1,1,1\rangle$

$f(x,y,z) = e^{xyz} + g(y,z)$

$f_y = ye^{xyz} + g_y(y,z) = ye^{xyz}$ $g(y,z) = 0$

$f_z = ze^{xyz} + h(z) = ze^{xyz}$ $h(z) = 0$

$f(1,1,1) - f(0,0,0) = \boxed{e-1}$

1b. $r(0) = \langle 0,1\rangle$ $r(\sqrt{\pi/2}) = \langle 1,0\rangle$

$f(x,y) = x^4y^2 + x + g(y)$

$f_y = 2x^4y + 1 = 2x^4y + g_y$ $g_y(y,z) = 1$

$f(x,y,z) = x^4y^2 + x + y$

$f(1,0) - f(0,1) = \boxed{0}$

Exam Problem 2. Most of the way was right, but I messed up the actual integration

2a. $\int_0^1 \int_0^{e^x} f(x,y)\,dy\,dx \rightarrow \int_0^1 \int_0^{\ln y} f(x,y)\,dx\,dy$

2b. $\int_0^{\pi} \int_0^{\sin x} f(x,y)\,dy\,dx \rightarrow \int_0^1 \int_0^{\sin^{-1} y} f(x,y)\,dx\,dy$

2c. $\int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy \rightarrow \int_1^e \int_0^{\ln x} f(x,y)\,dy\,dx$

Exam Problem 3. ✓

3a. $r(u,v) = \langle u^2, uv, v^2\rangle$ $r_u = \langle 2u, v, 0\rangle$ $r_v = \langle 0, u, 2v\rangle$

$r_u(1,2) \times r_v(1,2) = \langle 8,-4,1\rangle$

$8(x-1) - 4(y-2) + (z-4) = 0$

$8x - 8 - 4y + 8 + z - 4 \rightarrow \boxed{z = 4y - 8x + 4}$

3b. $r(u,v) = \langle u^3, v^3, -2uv\rangle$ $r_u = \langle 3u^2, 0, -2v\rangle$ $r_v = \langle 0, 3v^2, -2u\rangle$

$r_u(-1,1) \times r_v(-1,1) = \langle 6,-6,9\rangle$

$2(x+1) - 2(y+1) + 3(z-2) = 0$

$2x+2-2y-2+3z-6=0 \rightarrow \boxed{z = \frac{2}{3}y - \frac{2}{3}x + 2}$

Exam Problem 4. ✓

4a. $F =$ grad$F$, therefore $F(b) - f(a) = \int_C \nabla f \cdot dr$

$r(0) = \langle 0,0,0\rangle$ $r(3) = \langle 3, 9, 27\rangle$

$f(r(3)) - f(r(0)) = \boxed{\sin(3+9^2+27^3)}$

4b. $F =$ grad$F$, therefore $f(b) - f(a) = \int_C \nabla f \cdot dr$

$r(0) = \langle 0,1\rangle$ $r(1) = \langle 0,1\rangle$

$f(1) - f(0) = \boxed{0}$

Exam Problem 5. ✓

5a. $\int_R (x+y)(x^2+y^2+z^2)^2 \, dxdydz$

$\iiint \rho^4 (x+y) \, dxdydz \rightarrow \iiint \rho^7 \sin^2\phi (\sin\theta + \cos\theta) \, d\rho d\phi d\theta$

$\int_\pi^{2\pi} \int_{\frac{\pi}{2}}^{\pi} \int_0^1 \rho^7 \sin^2\phi (\sin\theta + \cos\theta) \, d\rho d\phi d\theta$

$\frac{1}{8} \int_\pi^{2\pi} \int_{\frac{\pi}{2}}^{\pi} \sin\phi \sin\theta + \sin\phi \cos\theta \, d\phi d\theta$

$\frac{1}{8} \int_\pi^{2\pi} \sin\theta + \cos\theta \, d\theta = \boxed{0}$

5b. $\int_R z(x^2+y^2+z^2) \, dxdydz \rightarrow \iiint \rho^5 \cos\phi \sin\phi \, d\rho d\phi d\theta$

$\int_0^\pi \int_0^{2\pi} \int_0^1 \rho^5 \cos\phi \sin\phi \, d\rho d\phi d\theta$

$\frac{1}{6} \int_0^\pi \int_0^{2\pi} \cos\phi \sin\phi \, d\phi d\theta \rightarrow \frac{1}{12} \int_0^\pi \int_0^{2\pi} \sin 2\phi \, d\phi d\theta$

$= \boxed{0}$

5c. $\int_R (z-x)\,dx\,dy\,dz \rightarrow \iiint (\rho\cos\phi - \rho\sin\phi\cos\theta)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \boxed{0}$

$\iiint \rho^3\sin\phi(\cos\phi - \sin\phi\cos\theta)\,d\rho\,d\phi\,d\theta$ cancel each other out $\rightarrow 0$

Full sphere means $0 \rightarrow 2\pi$

Exam Problem 6. ✓

6a. $\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y)\,dy\,dx \rightarrow \int_0^3\int_{\frac{\pi}{2}}^{\pi} r^3\cos^2\theta + r^2\sin\theta\,d\theta\,dr$

6b. $\int_0^4\int_{-\sqrt{16-x^2}}^{0} (x^2+y)\,dy\,dx \rightarrow \int_{\frac{3\pi}{2}}^{2\pi}\int_0^4 r^3\cos^2\theta + r^2\sin\theta\,dr\,d\theta$

6c. $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}}\int_{x}^{\sqrt{1-x^2}} (x^3+y^2)\,dy\,dx \rightarrow \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^1 r^4\cos^3\theta + r^3\sin^2\theta\,dr\,d\theta$

Exam Problem 7. ✓

7a. $\lim_{(x,y)\to(0,0)} \frac{x-1}{y-3} = \frac{0}{0}$ $\quad y = mx \quad \frac{x-1}{mx-3}$ $\quad$ lim DNE

7b. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{0}{0}$ Test values near $(0,0,.1)$ $\lim = 1$

Exam Problem 8. ✓

8a. $\langle 1-t\rangle(0,0,0) + \langle 1,1,-1\rangle t \quad r(t) = \langle t,t,-t\rangle \quad 0 \le t \le 1$

$r'(t) = \langle 1,1,-1\rangle \rightarrow \sqrt{3}$

$f(x,y,z) = xy^2 + yz^2 + z = 2t^3 - t$

$\sqrt{3}\int_0^1 2t^3 - t\,dt \rightarrow \sqrt{3}\left[\frac{1}{2}t^4 - \frac{1}{2}t^2\right]_0^1 = \boxed{0}$

8b. $r(t) = \langle r\cos t, r\sin t\rangle \quad r'(t) = \langle -\sin t, \cos t\rangle \quad 0 \le t \le \pi$

$\int_0^\pi \cos t + \sin t\,dt$

$\sin t - \cos t\Big|_0^\pi \rightarrow -\cos\pi + \cos 0 = \boxed{2}$

Exam Problem 9. Was doing correct way but messed up the bounds

9a. $F = \langle x+z, y+z, -x \rangle$ $P = x+z$, $Q = y+z$, $R = -x$

$\frac{\partial g}{\partial x} = -2x$, $\frac{\partial g}{\partial y} = -2y$

$\iint -(x+z)(-2x) - (y+z)(-2y) - x \, dA$

$\iint 2x^2 + 2xz + 2y^2 + 2yz - x \, dA$

$\iint 2x^2 + 18x - 2x^3 - 2xy^2 + 2y^2 + 18y - 2x^2y - 2y^3 - x \, dA$

$-\sqrt{9-x^2} \leq y \leq 0$ $-3 \leq x \leq 0$

Using Symbolab $\rightarrow$ $\boxed{-56.9827}$

9b. Integrand is same as 9a only bounds change

$0 \leq y \leq \sqrt{1-x^2}$ $0 \leq x \leq 1$

Using Symbolab $\rightarrow$ $\boxed{11.6521}$

Exam Problem 10. Everything was right, but I gave the wrong answer type.

10a. $\underbrace{yz = 2\lambda \quad xz = \lambda \quad xy = \lambda}$ $2x+y+z=4$

$2x = y = z$ $\rightarrow$ $y+y+y = 4$

$z = \frac{4}{3}$ $x = \frac{2}{3}$ $y = \frac{4}{3}$

$(\frac{4}{3})(\frac{4}{3})(\frac{2}{3}) \rightarrow \boxed{\frac{32}{27}}$

10b. $y^2z = 2\lambda$ $\underbrace{2xyz = \lambda \quad xy = \lambda}$ $2x+y+z=4$

$2xyz = xy$

$2z = 1$

$\frac{1}{2}y^2 = 2\lambda$

$\frac{1}{4}y^2 = 2xy$

$y = 4x$

$2x + 4x + \frac{1}{2} = 4$

$4x + 8x + 1 = 4$

$12x = 3$

$x = \frac{1}{4}$

$\boxed{(\frac{1}{4}, 1, \frac{1}{2})}$