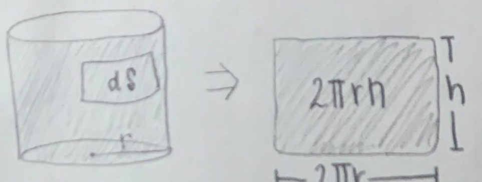
topics to review

- Surface integrals → converting to polar coordinates
- absolute max/min values
- local min, max + saddle points
- Equation for plane (containing 2 lines)
- ~~review derivatives of trig functions~~
- line integral
- F · dr along curve
- volume integrals
- Divergence Theorem

REVIEW TOPICS

SURFACE INTEGRALS

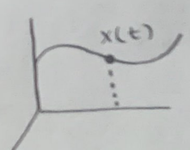


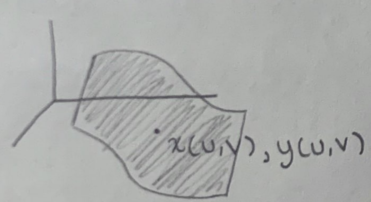
dS = tiny piece of surface

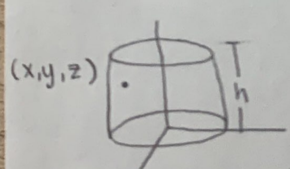
$$\underbrace{a(S)}_{\text{area of surface}} = \iint dS$$

Find area, excluding top and bottom

To find dS, we have to parametrize the cylinder.
↳ using (u,v)







$x^2+y^2=r^2$

parametrizing (x,y,z)

$x = r\cos\theta$

$y = r\sin\theta$

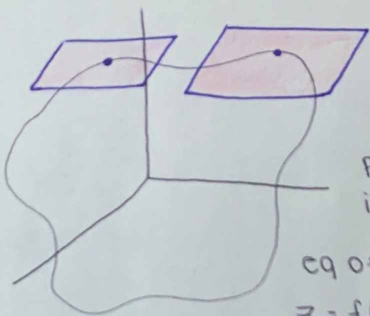
$x^2+y^2=r^2$

$z = z$

so that...

$= (r\cos\theta, r\sin\theta, z)$

OPTIMIZATION



critical point in 3D:
pt where tangent plane is parallel to xy plane. $\Rightarrow$ implies $f_x = 0$ and $f_y = 0$

eq of tangent plane:

$z = f(x_0, y_0) - f_x(P)x_0 - f_y(P)y_0$

where $P = (x_0, y_0) \rightarrow$ critical pt

$P(x,y)$ = critical point if...

① $P = (x,y)$ has undefined $f_x(P)$ or $f_y(P)$

② $f_x(P) = 0$ and $f_y(P) = 0$

GAUSS'/DIVERGENCE THEOREM

$$\iint_{\partial W} F \cdot dS = \iiint_W (\nabla \cdot F)\, dV = \iiint_W \operatorname{div}(\vec{F})\, dV$$

★Boundary of W is a <u>CLOSED</u> surface

FUNDAMENTAL THEOREM OF LINE INTEGRALS

$$\int_c F \cdot dr = f(b) - f(a)$$

where a and b are the start and end pts. (if given parametric eqs. plug in upper + lower bounds of t to obtain a and b).

Problem #1 (from Practice Final): Line Integral

$\int_C 7y\,dx + 3x\,dy$

Parametrize line integrals using "t" → do not mistake by using dr and dθ (that's for volume/area integrals)

Step #1: Parametrize using "t"

$x = r\cos t \Rightarrow x = 10\cos t$

$y = r\sin t \Rightarrow y = 10\sin t$ $\quad 0 \le t \le 2\pi$

$dx = -10\sin t$

$dy = 10\cos t$

Step #2: Rewrite equation in terms of "t"

$$\int_0^{2\pi} 7(10\sin t)(-10\sin t) + 3(10\cos t)(10\cos t)\,dt$$

$$\int_0^{2\pi} -700\sin^2 t + 300\cos^2 t\,dt$$

Apply double-angle formulas:

$$= -700\int_0^{2\pi} \frac{1-\cos(2t)}{2}\,dt + 300\int_0^{2\pi} \frac{1+\cos(2t)}{2}\,dt$$

$$= -700\left[\frac{1}{2}t - \frac{1}{4}\sin(2t)\right]\Big|_0^{2\pi} + 300\left[\frac{1}{2}t + \frac{1}{4}\sin(2t)\right]\Big|_0^{2\pi}$$

$$= -700[\pi - 0] + 300[\pi + 0] = -400\pi$$ for counter clockwise

For clockwise:

$-400\pi(-1) = \boxed{+400\pi}$

Problem #3: Absolute Max and Min

$f(x,y) = x^2y$

$\{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$

First, check critical points:

$f_x = 2xy = 0$

$f_y = x^2 = 0$

$x = 0 \Rightarrow 2(0)y = 0$

$y = 0$

critical pt: $(0,0)$

$f(0,0) = 0$

Absolute Min $= 0$

Absolute Max $= \frac{4}{27}$

Second, check boundaries:

$x = 0 \ldots$

$f(0,y) = 0$

$x = 1 \ldots$

$f(x,y) = x^2y$

$f(1,y) = y$

$y = 0 \ldots$

$f(x,0) = 0$

$f(x, 1-x) \ldots$

$= (x)^2(1-x) = x^2 - x^3$

↳ derivative to find x-value

$2x - 3x^2 = 0$

$2x(1 - \frac{3}{2}x) = 0 \Rightarrow x = \frac{2}{3}$

y at $x = 2/3 \ldots$

$f(\frac{2}{3}, y) = (\frac{2}{3})^2(1 - \frac{2}{3})$

$= \frac{4}{9}(1 - \frac{2}{3}) = \frac{4}{9} - \frac{8}{27}$

$= \frac{12}{27} - \frac{8}{27} = \frac{4}{27}$

Problem #4: Partial Derivatives

$f_{xxyz}(0,0,0) = ?$

$f(x,y,z) = \sin(x^2+y+z)$

$f_x = \cos(x^2+y+z)(2x)$

$f_{xx} = 2\cos(x^2+y+z) + 2x(-\sin(x^2+y+z)(2x))$

$= 2\cos(x^2+y+z) - 4x^2\sin(x^2+y+z)$

$f_{xxy} = -2\sin(x^2+y+z) - 4x^2\cos(x^2+y+z)$

$f_{xxyz} = -2\cos(x^2+y+z) + 4x^2\sin(x^2+y+z)$

$f_{xxyz}(0,0,0) = -2\cos(0) + 4(0)^2\sin(0) = \boxed{-2}$

Problem #6: Equation of Planes

$$\left.\begin{aligned} x &= 1+t \\ y &= 2+t \\ z &= 3+t \\ x &= -t \\ y &= 1+t \\ z &= 2+t \end{aligned}\right\} (-\infty < t < +\infty)$$

Step #1: Find direction vectors

From looking @ direction of t-var:

Line #1: $\langle 1,1,1 \rangle$

Line #2: $\langle -1,1,1 \rangle$

Step #2: Find normal vector (cross product of direction vectors

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = (1-1)\hat{i} - (1+1)\hat{j} + (1+1)\hat{k}$$

$= \langle 0,-2,2 \rangle = \langle a,b,c \rangle$

Step #3: Obtain a point and make equation:

$x(0) = -0 = 0$

$y(0) = 1+0 = 1$

$z = 2+0 = 2$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = d$

$0(x-0) + -2(y-1) + 2(z-2) = 0$

$-2y+2+2z-4 \rightarrow -2y+2z = 2 \rightarrow \boxed{-y+z=1}$

Problem #4: Partial Derivatives

$f_{xxyz}(0,0,0) = ?$

$f(x,y,z) = \sin(x^2+y+z)$

$f_x = \cos(x^2+y+z)(2x)$

$f_{xx} \; 2\cos(x^2+y+z) + 2x(-\sin(x^2+y+z)(2x))$

$= 2\cos(x^2+y+z) - 4x^2\sin(x^2+y+z)$

$f_{xxy} = -2\sin(x^2+y+z) - 4x^2\cos(x^2+y+z)$

$f_{xxyz} = -2\cos(x^2+y+z) + 4x^2\sin(x^2+y+z)$

$f_{xxyz}(0,0,0) = -2\cos(0) + 4(0)^2\sin(0) = \boxed{-2}$

Problem #6: Equation of Planes

$$\left.\begin{array}{l} x = 1+t \\ y = 2+t \\ z = 3+t \\ x = -t \\ y = 1+t \\ z = 2+t \end{array}\right\} (-\infty < t < +\infty)$$

Step #1: Find direction vectors From looking @ direction of t-var:

Line #1: $\langle 1,1,1\rangle$

Line #2: $\langle -1,1,1\rangle$

Step #2: Find normal vector (cross product of direction vectors

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = (1-1)\hat{i} - (1+1)\hat{j} + (1+1)\hat{k}$$

$$= \langle 0,-2,2\rangle = \langle a,b,c\rangle$$

Step #3: Obtain a point and make equation:

$x(0) = -0 = 0$ $\quad a(x-x_0) + b(y-y_0) + c(z-z_0) = d$

$y(0) = 1+0 = 1$ $\quad 0(x-0) + -2(y-1) + 2(z-2) = 0$

$z = 2+0 = 2$ $\quad -2y+2+2z-4 \to -2y+2z = 2 \to \boxed{-y+z=1}$

Problem #8: Line Integral

$\int_c (x+y+2z)\,dS \longrightarrow \int_0^1 (t+2t+2(2t))(3))dt$

$r(t) = \langle t, 2t, 2t \rangle$ $\quad 0 \le t \le 1$

$r'(t) = \langle 1, 2, 2 \rangle$

$\|r'(t)\| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$

$= \int_0^1 (21t)\,dt = \left[\frac{21}{2}t^2\right] = \boxed{\frac{21}{2}}$

Problem #10: Gauss' Theorem/Divergence Theorem

$\iint_S F \cdot dS$

$F = \langle x^2 + \sin(y+z),\ y^2 + xz^3,\ z^2 + e^{xy} \rangle$

$\{(x,y,z) \mid 0 \le x,y,z \le 1\}$

$\text{div}F = 2x + 0 + 2y + 0 + 2z = 2x + 2y + 2z$

$\int_0^1 \int_0^1 \int_0^1 (2x+2y+2z)\,dx\,dy\,dz$

inner int:

$\int_0^1 (2x+2y+2z)dx = [x^2 + 2xy + 2xz]\Big|_0^1 = 1 + 2y + 2z$

middle int:

$\int_0^1 (1+2y+2z)dy = [y + y^2 + 2zy]\Big|_0^1 = 1 + 1 + 2z = 2 + 2z$

outer int:

$\int_0^1 (2+2z)dz = [2z + z^2]\Big|_0^1 = 2 + 1 = \boxed{3}$

Question #11: Fundamental Theorem of Line Integrals

$f = \int P dx = \int Q dy = \int R dz = e^{2x+3y+4z}$

$x = t$

$y = 2t$

$z = t^2$

$0 \le t \le 1$

$$\int_c F \cdot dr = f(b) - f(a)$$

To find pts a and b, plug in upper and lower bounds of t $(0 \le t \le 1)$ into parametric eqs:

$\left.\begin{array}{l} x(0) = 0 \\ y(0) = 0 \\ z(0) = 0 \end{array}\right\} a = (0,0,0)$ $\left.\begin{array}{l} x(1) = 1 \\ y(1) = 2 \\ z(1) = 1 \end{array}\right\} b = (1,2,1)$

$$\int_c F \cdot dr = e^{2(1)+3(2)+4(1)} - e^0 = \boxed{e^{12} - 1}$$

Question #13: Volume Integral

multivariable calculus practice exam

problem 1.

$7y\,dx - 3x\,dy$

$x = 10\cos t$

$y = 10\sin t$

$x = r\cos t$

~~$x = 10\sin t$~~

$y = -r\sin t$

$0 \le t \le 2\pi$

$r^2 = x^2 + y^2 = \cancel{100}\ 10^2$

$\hookrightarrow 0 \le r \le \cancel{100}\ 10$

$x^2 + y^2 = r^2$

Therefore,

$r^2 = 100$

$r = 10$

$(r \ne 100!)$

convert to Polar coord:

$$\int_0^{2\pi}\int_0^{100} 7(-r\sin t)(-r\sin t) + 3(r\cos t)(-r\cos t)\,dr\,d\theta$$

First Integral:

$$\int_0^{100} 7r^2\sin^2 t \quad 3r^2\cos^2 t\,dr$$

$$\left[\frac{7}{3}r^3\sin^2 t - r^3\cos^2 t\right]\Big|_0^{100} = \frac{7(100)^3}{3}\sin^2 t - (100)^3\cos t$$

Outer Integral:

$$\Rightarrow \int_0^{2\pi} \frac{7(100^3)}{3}\sin^2 t - 100^3\cos t$$

$$\frac{7(100^3)}{3}\left(-\frac{\sin(2\pi)\cos(2\pi)}{2} + \pi\right) + 100^3\left[\sin(2\pi) - \sin(0)\right]$$

$$= \frac{7(100^3)}{3}(0 + \pi) + 0 = \cancel{\boxed{\frac{7(100^3)}{3}\pi}}$$

400π

CLOSE, minor but CRUCIAL error.

problem # 2.

equation of tangent plane:

$a(x - x_0) + b(y - y_0) + c(z - z_0) = d$

check if point is on plane:

$5 = (1)^2 + 3(1)(1) + (1)^2$

$5 = 5 \Rightarrow$ point is on the plane

$f_x = 2x + 3y$

$\hookrightarrow f_x(1,1,5) = 2(1) + 3(1) = 5$

$f_y = 3x + 2y$

$\hookrightarrow f_y(1,1,5) = 3(1) + 2(1) = 5$

Plane Tangent:

$z = 5 + 5(x - 1) + 5(y - 1)$

$z = \cancel{5} + 5x - \cancel{5} + 5y - 5$

$z = 5x + 5y - 5 \Rightarrow \boxed{z = 5(x + y - 1)}$

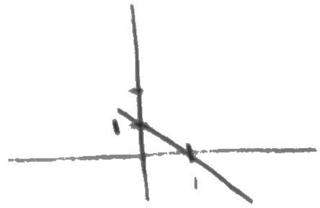
CORRECT!

+12

Problem #3

$f(x,y) = x^2 y$

$\{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$



$f_x \; 2xy$

$f_{xx} \; 2y$

$f_y = x^2$

$f_{yy} = 0$

$f_{xy} = 2x$

Find critical points.

$f_x = 0 = 2xy$

$y = 0$

$f_y = 0 = x^2$

$x = 0$

$f(0,0) = 0$

critical point at $(0,0)$

Testing Boundary Values:

$f(0,y) = 0^2(y) = 0$

$f(1,y) = 1^2 y = y = F(y)$

$F'(y) = 1$

$f(1,1) = (1)^2(1) = 1$

$f(x,0) = 0$

$f(x, 1-x) = x^2(1-x) = x^2 - x^3 = F(x)$

$F'(x) = \frac{1}{3}x^3 - \frac{1}{4}x^4 = 0$

$x^3(\frac{1}{3} - \frac{1}{4}x) \Rightarrow x = 0, \frac{4}{3}$

$f(\frac{4}{3}, 1 - \frac{4}{3}) = f(\frac{4}{3}, -\frac{1}{4}) \rightarrow$ not in boundaries

Absolute Max = 1 (at $(1,1)$)

Absolute Min = 0

NOPE.

completely redo this.

Problem #4.

$f(x,y,z) = \sin(x^2+y+z)$

$f_{xxyz}(0,0,0) = ?$

$f_x = \cos(x^2+y+z)(2x)$

$f_{xx} = -\sin(x^2+y+z)(2x)(2x)$

$f_{xxy} = -\cos(x^2+y+z)(4x^2)$

$f_{xxyz} = \sin(x^2+y+z)(4x^2)$

$\boxed{f_{xxyz}(0,0,0) = 0}$

Nope. :(

redo this.

Problem #5.

$$x + \cancel{x\frac{dz}{dy}} + z + \cancel{y\frac{dz}{dy}} + 2x^2yz^2 + \cancel{2x^2y^2z\frac{dz}{dy}} = 0$$

$$\frac{dz}{dy}(x+y+2x^2y^2z) = -x - z - 2x^2yz^2$$

$$\frac{dz}{dy} = \frac{-x-z-2x^2yz^2}{x+y+2x^2y^2z} \Rightarrow \frac{dz}{dy}(1,1,1) = \frac{-(1)-(1)-2(1)}{1+1+2(1)} = \frac{-3}{3} = -1$$

$\frac{-4}{4}$?

$$\boxed{\frac{dz}{dy}(1,1,1) = -1}$$

RIGHT ANSWER! +12

Problem #6.

$x = 1+t$

$y = 2+t$

$z = 3+t$

$(-\infty < t < +\infty)$

at $t=0$:

$x=1$

$y=2$

$z=3$

$x = -t$

$y = 1+t$

$z = 2+t$

$x=0$

$y=1$

$z=2$

Find cross product:
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix}$$

$= (4-3)\hat{i} - (2-0)\hat{j} + (1-0)\hat{k}$

$= 1\hat{i}\ \cancel{+0}\,\hat{j}\ \overset{-2}{}\ + \cancel{1}\hat{k}\ \overset{1}{} = \cancel{\langle\ \rangle}$ $\langle 1, -2, 1\rangle$

Equation:

$1(x-1) + \overset{-2}{\cancel{0}}(y-2) + \overset{1}{\cancel{0}}(z-3) = 0$

$x - 1 = 0$

$\boxed{x = 1}$

$x \cancel{-1} + (-2y \cancel{+4}) + z \cancel{-3} = 0$

$x - 2y + z = 0$

WRONG. redo.

Problem #7.

$a(t) = \langle -4\sin(2t), -4\cos(2t), 9e^{3t} \rangle$

$v(0) = \langle 2, 0, 3 \rangle$

$r(0) = \langle 0, 1, 1 \rangle$

$v(t) = \int a(t)\,dt + C$

$= \langle 2\cos(2t), -2\sin(2t), 3e^{3t} \rangle$

since $v(0) = \langle 2, 0, 3 \rangle \ldots$

$\boxed{v(t) = \langle 2\cos(2t), -2\sin(2t), 3e^{3t} \rangle}$

$r(t) = \int v(t)\,dt + C$

$= \langle \sin(2t), \cos(2t), e^{3t} \rangle$

since $r(0) = \langle 0, 1, 1 \rangle$

$\boxed{r(t) = \langle \sin(2t), \cos(2t), e^{3t} \rangle}$

$r(\frac{\pi}{4}) = \langle \sin(\frac{2\pi}{4}), \cos(\frac{2\pi}{4}), e^{3(\frac{\pi}{4})} \rangle$

$\boxed{r(\frac{\pi}{4}) = \langle 1, 0, e^{\frac{3\pi}{4}} \rangle}$

yes! +12

Problem 8.

$\int_c (x + y + 2z)\,ds = \int_0^1 (t + 2t + 2(2t)) \sqrt{(1)^2 + (2)^2}\,dt$

$r(t) = \langle t, 2t, 2t \rangle \quad 0 \le t \le 1$

$= \int_0^1 7t\sqrt{5}\,dt = 7\sqrt{5} \int_0^1 t\,dt = 7\sqrt{5}\left[\frac{1}{2}t^2\right]\Big|_0^1 = \boxed{\frac{7\sqrt{5}}{2}}$

$\frac{21}{2}$

Problem 9.

$\lim_{(x,y,z)\to(1,1,1)} f(x,y,z) = 1 \quad , \lim_{(x,y,z)\to(1,1,1)} g(x,y,z) = 2$

compute:

$\lim_{(x,y,z)\to(1,1,1)} \sin\left(\frac{\pi}{3} \overbrace{f(x,y,z)}^{1}\right) \cos\left(\frac{\pi}{4} \overbrace{g(x,y,z)}^{2}\right)$

$\hookrightarrow \lim_{(x,y,z)\to(1,1,1)} \sin(\frac{\pi}{3}) \cos(\frac{\pi}{2}) = \boxed{0}$

yes! +12

Problem #10.

$\iint_S F \cdot dS$

$F = \langle x^2 + \sin(y+z),\ y^2 + xz^3,\ z^2 + e^{xy} \rangle$

$\{(x,y,z) \mid 0 \le x,y,z \le 1\}$

w/ normal pointing outward

$\iint_S F \cdot dS = \iiint_W (\nabla \cdot F)\, dV$

$\nabla F = \langle 2x, 2y, 2z \rangle$

$\int_0^1 \int_0^1 \int_0^1 (2x + 2y + 2z)\, dx\, dy\, dz$

$\int_0^1 (2x + 2y + 2z)\, dx$

$= [x^2 + 2yx + 2zx] \Big|_0^1 = [1 + 2y + 2z]$

$\int_0^1 (1 + 2y + 2z)\, dy = [y + y^2 + 2zy] \Big|_0^1$

$= [2 + 2z]$

$\int_0^1 2 + 2z\, dz = [2z + z^2] \Big|_0^1 = \boxed{3}$

yes!

+12

Problem 11

$F(x,y,z) = \langle 2e^{2x+3y+4z},\ 3e^{2x+3y+4z},\ 4e^{2x+3y+4z} \rangle$

C: $x = t$
$y = 2t$
$z = t^2$

$0 \le t \le 1$

$f = \int P\, dx = \int 2e^{2x+3y+4z}\, dx = e^{2x+3y+4z}$

$f = \int Q\, dy = \int 3e^{2x+3y+4z}\, dy = e^{2x+3y+4z}$

$f = \int R\, dz = \int 4e^{2x+3y+4z}\, dz = e^{2x+3y+4z}$

$\boxed{f = e^{2x+3y+4z}}$

(problem 11. cont'd)

$\int_c F \cdot dr$

$F(x,y,z) = \langle 2e^{2x+3y+4z}, 3e^{2x+3y+4z}, 4e^{2x+3y+4z} \rangle$

C: $x = t$

$y = 2t$ $\qquad \int_c \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

$z = t^2$

$0 \leq t \leq 1$ $\qquad = \int_0^1 \langle 2e^{2t+6t+4t^2}, 3e^{3t+6t+4t^2}, 4e^{3t+6t+4t^2} \rangle \cdot \langle 1, 2, 2t \rangle$

$r(t) = \langle t, 2t, t^2 \rangle$ $\qquad = \int_0^1 2e^{2t+6t+4t^2} + 6e^{2t+6t+4t^2} + 8te^{2t+6t+4t^2}\, dt$

$r'(t) = \langle 1, 2, 2t \rangle$

$$= \int_0^1 \left[8e^{8t+4t^2} + 8te^{8t+4t^2} \right] dt$$

$$= 8\int_0^1 e^{8t+4t^2}\, dt + 8\int_0^1 te^{8t+4t^2}\, dt$$

$$= \frac{8}{8+8t} e^{8t+4t^2}$$

↳ input into maple outputs imaginary #s

↳ can't be a negative #

NOPE.

(sort of)

redo!

Problem #12.

$\int_c 5y\,dx + 5x\,dy + 6z\,dz$ $= \int_0^1 \underbrace{(5t)(2t)}_{10t^2} + \underbrace{(5t^2)(1)}_{+\ 5t^2} + \underbrace{6t^2(2t)}_{12t^3}\ dt$

$C: x = t^2, y = t, z = t^2$

$0 \le t \le 1$

$= \int_0^1 15t^2 + 12t^3\ dt$

$x'(t) = 2t$

$= \left[5t^3 + 3t^4\right]\Big|_0^1 = \boxed{8}$

$y'(t) = 1$

$z'(t) = 2t$

yes! +12

Problem #13.

$\iiint_E \frac{1}{\sqrt{x^2+y^2+z^2}}\,dV$ $= \int_0^{10}\int_0^{\sqrt{100-x^2}}\int_0^{\sqrt{100-x^2-y^2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\,dz\,dy\,dx$

$\{(x,y,z) \mid x^2+y^2+z^2 \le 100, z < 0\}$ $\Rightarrow \int_0^{\sqrt{100-x^2-y^2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\,dz$

$x^2+y^2+z^2 \le 100$

$z \le \sqrt{100-x^2-y^2}$ $= \frac{-2z}{(x^2+y^2+z^2)^2}\Bigg]\Bigg|_0^{\sqrt{100-x^2-y^2}}$

when $z = 0 \ldots$

$x^2+y^2 \le 100$ $= \frac{-2\sqrt{100-x^2-y^2}}{(\cancel{x^2}+\cancel{y^2}+100-\cancel{x^2}-\cancel{y^2})^2}$

$y \le \sqrt{100-x^2}$

$0 \le y \le \sqrt{100-x^2}$

$= \int_0^{\sqrt{100-x^2}} \frac{-2\sqrt{100-x^2-y^2}}{(100)^2}\,dy =$

$0 \le x \le 10$

$$\int_0^{\sqrt{100-x^2}} \frac{\sqrt{100-x^2-y^2}}{5000}\, dy \ \exists \text{ using maple:}$$

$\cancel{u = 100 - x^2 - y^2}$

$\cancel{du = -2x}$

$$= \frac{y}{5000\sqrt{-x^2-y^2+100}} \Bigg|_0^{\sqrt{100-x^2}}$$

$$= \frac{\sqrt{100-x^2}}{5000\sqrt{-\cancel{x^2}-(\cancel{100}-\cancel{x^2})+\cancel{100}}}$$

$$= \frac{\sqrt{100-x^2}}{5000}$$

$$\int_0^{10} \frac{\sqrt{100-x^2}}{5000} \ \exists \text{ using maple:}$$

$$= \frac{-x}{5000\sqrt{-x^2+100}} \Bigg|_0^{10} = \frac{-10}{5000\sqrt{-100+100}} = 0 \quad ?$$

Nope.

redo.

Problem #16:

Find local min and max:
(and saddle pts)

$f(x,y) = x^3 + y^2 - 6xy$

$f_x = 3x^2 - 6y = 0$

$f_y = 2y - 6x = 0$

$2y = 6x$

$y = 3x$

$f_x = 3x^2 - 3x = 0$

$3x(x-1) = 0$

$\Rightarrow x = 0, +1$

if $x = 0$ and $+1$...

$y = 3(0) = 0$

$y = 3(1) = 3$

critical points:

$(0,0)$

$(1,3)$

~~f(0,0)~~

$f_{xx} = 6x$

$f_{yy} = 2$

$f_{xy} = -6$

$D = (6x)(2) - (-6)^2 = 12x - 36$

$D(0,0) = 12(0) - 36 = -36$

$D(1,3) = 12(1) - 36 = -24$

$f_{xx}(0,0) = 6(0) = 0$

$f_{xx}(1,3) = 6(1) = 6$

since $D < 0 \Rightarrow (0,0)$ and $(1,3)$ are saddle points

redo.

Problem #14

$$\int_0^1 \int_0^w \int_0^z \int_0^y 360x \, dx \, dy \, dz \, dw$$

inner int: $\int_0^y 360x\,dx = [180x^2]|_0^y = 180y^2$

middle int: $\int_0^z 180y^2 dy = 60y^3]_0^z = 60z^3$

middle int #2 $\int_0^w 60z^3 dz = [15z^4]_0^w = 15w^4$

outter int: $\int_0^1 15w^4 dw = 3w^5|_0^1 = \boxed{3}$ yes! +12

Problem #15:

$$\begin{vmatrix} \frac{dx}{du} & \frac{dy}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} \end{vmatrix} \Rightarrow \begin{vmatrix} 3\cos(2u+v)(2) & 1-\sin(u+v) \\ 3\cos(2u+v)(1) & 1-\sin(u+v) \end{vmatrix}$$

$= (6\cos(2u+v))(1-\sin(u+v)) - (3\cos(2u+v))(1-\sin(u+v))$

@ $(u,v) = (0,0)$

$= (6\cos(0))(1-\sin(0)) - (3\cos(0))(1-\sin(0))$

$= (6)(1-0) - 3(1-0) = 6-3 = \boxed{3}$ yes! +12

SCII WORKSHEET

Problem #1.

Explanation - I got this problem wrong because I did not apply the Fundamental Theorem of Line Integrals.

Problem 1b.

apply fundamental theorem of line integrals:

$\int_c F \cdot dr = f(b) - f(a)$ where $F = \nabla f$

$\int_c (4x^3y^2 + 1)\,dx + (2x^4y + 1)\,dy \longrightarrow f = \int P\,dx = \int Q\,dy$

$r(t) = \langle \sin(t^2), \cos(t^2) \rangle$, $0 \le t \le \sqrt{\frac{\pi}{2}}$

$\cancel{dx = \cos(t^2)}$

$\cancel{dy = \sin(t^2)}$

$\Rightarrow \int P\,dx = \int (4x^3y^2 + 1)\,dx$

$= x^4y^2 + x$

$\Rightarrow \int Q\,dy = \int (2x^4y + 1)\,dy$

$= x^4y^2 + y$

so that...

$\boxed{f = x^4y^2 + x + y}$

Apply Fundamental Theorem of Line Integrals:

$r(0) = \langle \sin(0), \cos(0) \rangle = (0, 1)$

$r(\frac{\pi}{2}) = \langle \sin(\frac{\pi}{2}), \cos(\frac{\pi}{2}) \rangle = (1, 0)$

$\Rightarrow f(1,0) - f(0,1) = [(1)^4(0)^2 + 1 + 0] - [(0)^4(1)^2 + 0 + 1] = 1 - 1 = \boxed{0}$

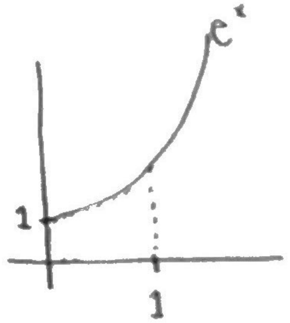
Answer

Problem #2.

Problem 2a.

$$\int_0^1 \int_0^{e^x} f(x,y)\,dy\,dx$$

$$\boxed{\int_0^e \int_0^{\ln(y)} f(x,y)\,dx\,dy}$$

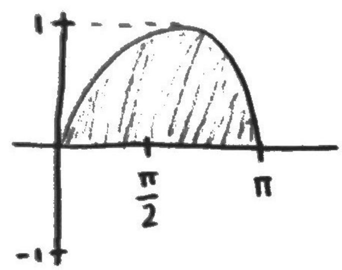


$y = e^x$

$\ln(y) = x$

Problem 2b

$$\int_0^{\pi} \int_0^{\sin x} f(x,y)\,dy\,dx$$



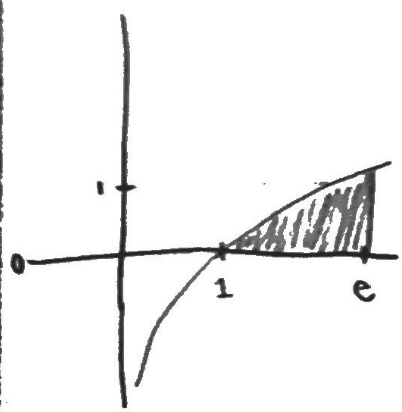
$y = \sin x$

$\sin^{-1}(y) = x$

$$\boxed{\int_0^1 \int_0^{\sin^{-1}(y)} f(x,y)\,dx\,dy}$$

Problem 2c.

$$\boxed{\int_1^e \int_0^{\ln(x)} f(x,y)\,dy\,dx}$$



$x = e^y$

$\ln(x) = y$

Problem #3.

Problem 3a.

$\vec{r} = \langle \;\; 2 \;\; 6 \rangle$

$x(u,v) = u^2$

$y(u,v) = uv$

$z(u,v) = v^2$

$\infty < u < \infty$

$-\infty < v < \infty$

Find position vectors r_u and r_v:

- $r_u = \langle 2u, v, 0 \rangle$
- $r_v = \langle 0, u, 2v \rangle$

$r_u \times r_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2v & v & 0 \\ 0 & u & 2v \end{vmatrix} = (2v)\hat{i} - (0 - 4uv)\hat{j} + (2vu - v)\hat{k}$

$= \langle 2v, -4uv, 2vu - v \rangle$

@ $(u,v) = (1,2)$

$\Rightarrow \langle 2(2), -4(1)(2), 2(1)(2), -2 \rangle$

$= \langle 4, -8, 2 \rangle$

Eq $4(x-1) - 8(y-2) + 2(z-4) = 0$

$4x - 4 - 8y + 16 + 2z - 8 = 0$

$4x - 8y + 2z + 4 = 0$

$2z = 8y - 4x - 4$

$z = 4y - 2x - 2$

Problem #4:

Problem 4a.

$r(0) = \langle 0, 0, 0 \rangle$

$\vec{r}(3) = \langle 3, 9, 27 \rangle$

$\Rightarrow \sin(3 + 9^2 + 27^2) - \sin(0) = \sin(813)$ using Fundamental Theorem of Line Integrals

Problem #5:

Explanation: I did not use spherical coordinates to solve on the exam.

$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$$\int_R (x+y)(x^2+y^2+z^2)^2 \, dx\,dy\,dz$$

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$x^2+y^2+z^2 = \rho^2$

$\rho = 1$

$$\int_R (\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta)(\rho^2)^2 \, d\rho\, d\theta, d\phi$$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^5\sin\phi\cos\theta + \rho^5\sin\phi\sin\theta \, d\rho\, d\theta\, d\phi$$

inner int:

$$\left[\frac{1}{6}\rho^6\sin\phi\cos\theta + \frac{1}{6}\rho^6\sin\phi\sin\theta\right]\Big|_0^1 = \frac{1}{6}\sin\phi\cos\theta + \frac{1}{6}\sin\phi\sin\theta$$

middle int:

$$\int_{\frac{3\pi}{2}}^{2\pi}\frac{1}{6}\sin\phi\cos\theta + \frac{1}{6}\sin\phi\sin\theta \, d\theta = \left[\frac{1}{6}\sin\phi\sin\theta - \frac{1}{6}\sin\phi\cos\theta\right]\Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$= \left[0 - \frac{1}{6}\sin\phi\right] - \left[0 - \frac{1}{6}\sin\phi\right] = 0$$

outer int

$$\int_{\frac{\pi}{2}}^{\pi} d\phi = 0$$

☆ Be careful figuring out boundaries

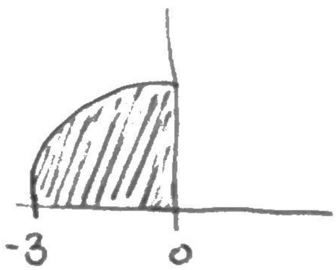
Problem #6:

Problem 6a.

$$\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y)\,dy\,dx \qquad \int_{\frac{\pi}{2}}^{\pi}\int_{0}^{3} (r^2\cos^2\theta + r\sin\theta)\,dr\,d\theta$$

$x = r\cos\theta$

$y = r\sin\theta$



$r = 3$

$\frac{\pi}{2} < \theta < \pi$

Problem #7:

problem 7a.

$$\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3} = \lim_{(x\to 1)} \frac{cx-c+3-1}{cx-c+3-3} = \lim_{(x\to 1)} \frac{cx-c+2}{cx-c}$$

$(y-3) = c(x-1)$

$y = cx - c + 3$

cannot cancel out, and since problem is reliant on slope c, the problem cannot be solved

Problem #8:

$(0,0,0)$

$(1,1,-1)$

$f(x,y,z) = xy^2 + yz + z$

~~$r(t) = \langle \cos t, \sin t \rangle$~~

$\int_0^{\pi}\int_0^1 \cos t + \sin t \, dr\, d\theta = [\sin t - \cos t]\Big|_0^1$

$x = (1-t)(0) + t(1)$

$x = t$

$y = (1-t)(0) + t(1)$

$y = t$

$z = (1-t)(0) + t(-1)$

$z = -t$

$r(t) = \langle t, t, -t \rangle$

$0 \le t \le 1$

$\|r'(t)\| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$

$\int_0^1 (t^3 - t^2 - t)(\sqrt{3})\,dt$

Problem 9:

$F = \langle x+z, y+z, -x \rangle$

$z = 9 - x^2 - y^2$

$z_x = -2x$

$z_y = -2y$

$\iint_D -Pg_x - Qg_y + R \, dA$

$\iint_D -(x+z)(-2x) - (y+z)(-2y) - x \, dA$

$\iint_D -(-2x^2 - 2xz) - (-2y^2 - 2yz) - x \, dA$

$\iint_D 2x^2 + 2xz + 2y^2 + 2yz - x$

~~$x = r\cos\theta$~~

~~$y = r\sin\theta$~~

$\int_0^1\int_0^1 (2x^2 + 2x(9 - x^2 - y^2) + 2(y)^2 + 2y(9 - x^2 - y^2) - x \, dx\, dy$

$= \int_0^1\int_0^1 2x^2 + 18x - 2x^3$