Second Chance Club 2 Worksheet

Exam Problem 1: I got this correct

1a. $\text{Curl}(F) = i(xz^2e^{xyz} - xy^2e^{xyz})$ etc., not zero, not conservative

$x(t) = t \quad x'(t) = 1$

$y(t) = t^2 \quad y'(t) = 2t$

$z(t) = t^3 \quad z'(t) = 3t^2$

$$\int_0^1 te^{t^6} + 2t^3e^{t^6} + 3t^5e^{t^6}\,dt$$

$$= \int_0^1 e^{t^6}(t + 2t^3 + 3t^5)\,dt$$

$$\boxed{I = 2.31129}$$

1b.

$\frac{\partial}{\partial y} 4x^3y^2 + 1 = 8x^3y$ ✓ Conservative

$\frac{\partial}{\partial x} 2x^4y + 1 = 8x^3y$ on inspection: $f(x,y) = x^4y^2 + x + y$

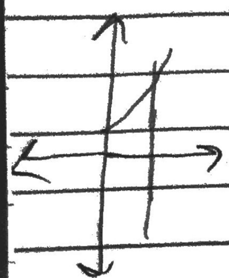
$r(\sqrt{\pi/2}) = \langle 1,0\rangle \quad f(r(\sqrt{\pi/2})) - f(r(0))$

$r(0) = \langle 0,1\rangle$

$$= 1 - 1 = \boxed{0}$$

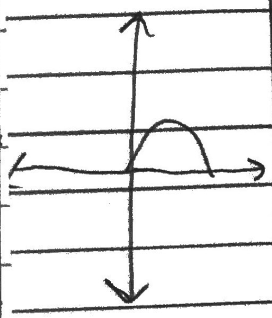
Vector is conservative so fundamental theorem of line integrals can be used

Exam Problem 2: I took an integral incorrectly. I can avoid this by checking my work more throroughly



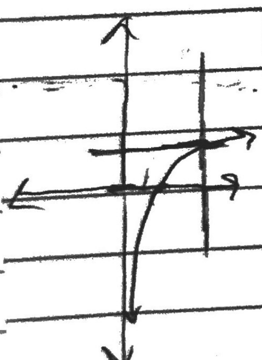
2a. $\int_0^1 \int_0^{e^x} f(x,y)\,dy\,dx$

$\boxed{= \int_0^e \int_{\ln(y)}^1 f(x,y)\,dx\,dy}$



2b. $\int_0^{\pi} \int_0^{\sin x} f(x,y)\,dy\,dx$

$\boxed{= \int_0^{\sqrt{2}/2} \int_{\sin^{-1}(y)}^{\pi} f(x,y)\,dx\,dy}$



2c. $\int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy$

$\boxed{= \int_0^e \int_0^{\ln(x)} f(x,y)\,dy\,dx}$

Exam Problem 3: I got this right

3a. @ point (1,2,4) $u=1$, $v=2$

$r_u = \langle 2u, v, 0 \rangle = \langle 2, 2, 0 \rangle$ $r_u \times r_v = \langle 8, -8, 2 \rangle$
$r_v = \langle 0, u, 2v \rangle = \langle 0, 1, 4 \rangle$

$8(x-1) - 8(y-2) + 2(z-4) = 0$

$4(x-1) - 4(y-2) + z - 4 = 0$ $\boxed{z = -4x + 4y}$
$4x - 4 - 4y + 8 + z - 4 = 0$
$4x - 4y + z = 0$

3b. @ point$(-1,-1,2)$

$u^3 = -1$, $u = -1$
$v^3 = -1$, $v = -1$
$2 = (-2)(-1)(-1)$
$2 \neq -2$

The point $(-1,-1,2)$ is not on the given surface.

Exam Problem 4: I got this right

4a. This problem calls for the fundamental theorem of line integrals as the vector is conservative

$r(3) = \langle 3, 9, 27 \rangle$ $f(r(3)) - f(r(0))$
$r(0) = \langle 0, 0, 0 \rangle$

$= \sin(19767) - \sin(0)$
$\boxed{= \sin(19767)}$

4b. This problem uses the fundamental theorem of line integrals as the vector is conservative

$r(\pi) = \langle 0, -1 \rangle$ $\quad f(r(\pi)) - f(r(0))$

$r(0) = \langle 0, 1 \rangle$

$= e^{\cos(0) + 3\sin(-1)} - e^{\cos(0) + 3\sin(1)}$

$$\boxed{= e^{3\sin(-1) + 1} - e^{3\sin(1) + 1}}$$

Exam Problem 5: I got this right

5a.

$0 \le \rho \le 1$

$\frac{3\pi}{2} \le \theta \le 2\pi$

$\frac{\pi}{2} \le \varphi \le \pi$

$$\int_{\frac{\pi}{2}}^{\pi} \int_{\frac{3\pi}{2}}^{2\pi} \int_{0}^{1} \rho^2 \sin^2\varphi (\cos\theta + \sin\theta)\rho^5 \, d\rho \, d\theta \, d\varphi$$

$$= \int_0^1 \rho^7 d\rho \int_{\frac{3\pi}{2}}^{2\pi} \cos\theta + \sin\theta \, d\theta \int_{\frac{\pi}{2}}^{\pi} \sin^2\varphi \, d\varphi$$

$$= \left(\frac{1}{7}\right)\left(\sin\theta - \cos\theta \Big|_{\frac{3\pi}{2}}^{2\pi}\right)\left(-\cos\varphi \Big|_{\frac{\pi}{2}}^{\pi}\right)$$

$$= \left(\frac{1}{7}\right)\left((0-1) - (-1-0)\right)(-1 - 0)$$

$$\left(\frac{1}{7}\right)(0)(-1) = \boxed{0}$$

5b.

$0 \le \rho \le 1$
$\pi \le \theta \le 2\pi$
$0 \le \varphi \le \pi$

$$\int_0^{\pi}\int_{\pi}^{2\pi}\int_0^{1} \rho\cos\varphi\, \rho^2 \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$= \int_0^1 \rho^5 d\rho \int_{\pi}^{2\pi} d\theta \int_0^{\pi} \sin\varphi\cos\varphi\, d\varphi$$

$$= \left(\frac{1}{6}\right)(\pi) \int_0^{\pi} \frac{\sin 2\varphi}{2}\, d\varphi$$

$$= \left(\frac{\pi}{12}\right)\left(\frac{-\cos 2\varphi}{2} \Big|_0^{\pi} \right)$$

$$= \left(\frac{\pi}{24}\right)(-1+1) = \boxed{0}$$

5c.

$0 \le \rho \le \sqrt{8}$
$0 \le \theta \le 2\pi$
$0 \le \varphi \le \pi$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{\sqrt{8}} (\rho\cos\varphi - \rho\sin\varphi\cos\theta)\rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$= \int_0^{\pi}\int_0^{2\pi}\int_0^{\sqrt{8}} \rho^3 (\sin\varphi\cos\varphi - \sin^2\varphi\cos\theta)\, d\rho\, d\theta\, d\varphi$$

$$= 64 \int_0^{\pi}\int_0^{2\pi} \sin 2\varphi - \sin^2\varphi\cos\theta \, d\theta\, d\varphi$$

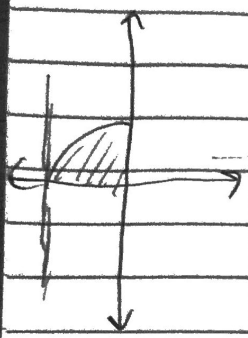
$$= 64 \int_0^{\pi} 2\pi \sin 2\varphi - \sin^2\varphi \left(\sin\theta \Big|_0^{2\pi} \right) d\varphi$$

$$= 64 \int_0^{\pi} 2\pi \sin 2\varphi - \sin^2\varphi (0 - 0)\, d\varphi$$

$$= 128\pi \int_0^{\pi} \sin 2\varphi \, d\varphi$$

$$\cancel{= 128\pi} \left(\frac{-\cos 2\varphi}{2} \Big|_0^{\pi} \right) = 64\pi(-1+1) = \boxed{0}$$

Exam Problem 6: I got this right

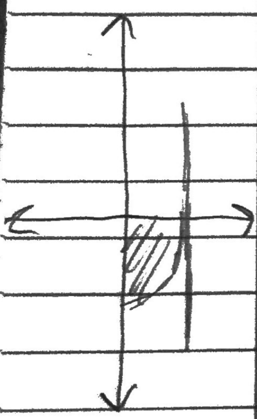


6a. $0 \le r \le 3$

$\frac{\pi}{2} \le \theta \le \pi$

$$\int_{\pi/2}^{\pi}\int_0^3 (r^2)^2 r \, dr \, d\theta$$

$$\boxed{= \int_{\pi/2}^{\pi}\int_0^3 r^8 \, dr \, d\theta}$$



6b.

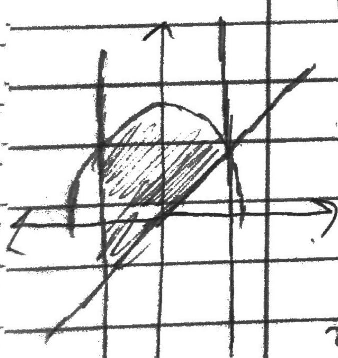
$0 \le r \le 4$

$\frac{3\pi}{2} \le \theta \le 2\pi$

$$\int_{3\pi/2}^{2\pi}\int_0^4 (r^2\cos^2\theta + \sin\theta) r \, dr \, d\theta$$

$$\boxed{= \int_{3\pi/2}^{2\pi}\int_0^4 r^3\cos^2\theta + r^2\sin\theta \, dr \, d\theta}$$

6c.



Integral 1:

$0 \le r \le 1$

$\frac{\pi}{4} \le \theta \le 3\pi/4$

Integral 2:

$0 \le r \le -\sqrt{2}/2 \sec\theta$

$\frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

$$\int_{\pi/4}^{3\pi/4}\int_0^1 r^4\cos^3\theta + r^3\sin^2\theta \, dr \, d\theta$$

$$+ \int_{3\pi/4}^{5\pi/4}\int_0^{-\frac{\sqrt{2}}{2}\sec\theta} r^4\cos^3\theta + r^3\sin^2\theta \, dr$$

$r\cos\theta = -\frac{\sqrt{2}}{2}$

$r = -\frac{\sqrt{2}}{2}\sec\theta$

Exam Problem 7: I didn't know how to do 7d., so I got it wrong. Practicing limit problems will help me learn how to do them for the future.

7a. $\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3}$

From x-axis $= (x,0)$ $\frac{x-1}{-3} = 0$

From line $y = cx$ (x,cx) $\frac{x-1}{cx-3} = \frac{0}{c-3} = 0$

$\therefore \boxed{\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3} = 0}$

7b. $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z}$

From x axis $(x,0,0)$ $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{x}{2x} = \frac{1}{2}$

From y axis $(0,y,0)$ $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{y}{y} = 1$

There is no agreement, so the limit does not exist! DNE

Exam Problem 8: I got this right

8a. $\int_C f\,ds = \int_0^1 t^3 + t^3 - t\,dt$

$x(t) = t$
$y(t) = t$
$z(t) = -t$

$= \int_0^1 2t^3 - t\,dt$

$= \frac{t^4}{2} - \frac{t^2}{2} \Big|_0^1$

$= \frac{1}{2} - \frac{1}{2} = \boxed{0}$

8b. $\int_C f\,ds = \int_0^{\pi} \cos t + \sin t\,dt$

$x(t) = \cos t$
$y(t) = \sin t$

$= \sin t - \cos t \Big|_0^{\pi}$

$= (0 - (-1)) - (0 - 1)$

$= 1 + 1 = \boxed{2}$

Exam Problem 9: I did not have the correct limits of integration. To prevent this from occuring again I can read the problem more carefully to recognize what the correct xy projection would be

9a.

$$\iint_S F \cdot dS = \iint (x+9-x^2-y^2)(2x) + (y+9-x^2-y^2)(2y) - x \, dA$$

$$= \int_{-3}^{0}\int_{-\sqrt{9-x^2}}^{0} (x+9-x^2-y^2)(2x) + (y+9-x^2-y^2)(2y) - x \, dy \, dx$$

$$\boxed{= -56.9827}$$

9b.

$$\iint_S F \cdot dS = \iint (x+9-x^2-y^2)(2x) + (y+9-x^2-y^2)(2y) - x \, dA$$

$$= \int_0^1\int_0^1 (x+9-x^2-y^2)(2x) + (y+9-x^2-y^2)(2y) - x \, dy \, dx$$

$$= 17.1667(-1) = \boxed{-17.1667}$$

Exam Problem 10: I answered the wrong question. I can fix this by reading the question more carefully next time, and making sure I'm answering the right question.

10a. $\nabla f = \lambda \nabla g$

$\langle yz, xz, xy \rangle = \lambda \langle 2, 1, 1 \rangle$

$yz = 2\lambda$ $\quad xz = xy \quad \frac{y}{x} = 2$

$xz = \lambda \quad y = z \quad y = 2x$

$xy = \lambda \quad 2x = y = z$

$2x + y + z = 4$

$6x = 4$

$x = 2/3 \quad y = 1/3 \quad z = 1/3$

$f(2/3, 1/3, 1/3) = 2/27$ $\boxed{\text{Max: } 2/27}$

10b.

$\nabla f = \lambda \nabla g$

$\langle y^2 z, 2xyz, xy^2 \rangle = \lambda \langle 2, 1, 1 \rangle$

$y^2 z = 2\lambda \qquad y^2 z = 2\lambda \qquad \frac{y}{z} = 2$

$2xyz = \lambda \rightarrow yz^2 = \lambda \qquad y = 2z$

$xy^2 = \lambda$

$2x + y + z = 4$

$\frac{z}{y} = 2 \qquad z + 2z + z = 4$

$z = 2x \quad \frac{z}{2} = x \qquad z = 1, y = 2, x = 1/2$

$\boxed{\text{Point: } (1/2, 2, 1)}$