MATH 251 (22,23,24), Dr. Z. Second Chance Club for Exam 2 Worksheet

Due Friday, Dec. 13, 2020, 8:00pm. Email an attachment called scc2FirstLast.pdf to DrZcalc3@gmail.com Subject: scc2

Make sure that you have the posted solutions:

http://www.math.rutgers.edu/~zeilberg/calc3NNN/mt2S.pdf

and understand **each** problem.

In this worksheet, you are supposed to state what was your error for each of the exam questions (if you made an error) and say how to avoid it in the future. Then, regardless of whether you got it right or wrong do the similar problems given.

Done

Exam Problem 1. (10 pts.) Compute the line integral

$$\int_C yz\,dx + (xz+z)\,dy + (xy+y+1)\,dz \quad ,$$

over the path

$$\mathbf{r}(t) = \langle e^{t^3}, t^2 e^{t^4}, t e^{t^7} \rangle \quad , \quad 0 \le t \le 1 \quad .$$

Curl = 0

Explain!

:

Here is what I did wrong (if applicable):

Problem 1a. Compute the line integral Curl ≠ 0

$$\int_C x e^{xyz}\,dx + y e^{xyz}\,dy + z e^{xyz}\,dz \quad , = \int_0^1 t e^{t^3} dt + t^2 e^{t^3}(2t)dt + t^3 e^{t^3}(3t^2) dt$$

$$= \int_0^1 e^{t^3} (t + 2t^3 + 3t^5)\,dt$$

over the path

$e^{xyz} = e^{t^3}$

$$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \quad , \quad 0 \le t \le 1 \quad .$$

Explain!

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

Problem 1b. Compute the line integral

Curl = 0

$$\int_C \underbrace{(4x^3y^2+1)}_{f_x}\,dx + \underbrace{(2x^4y+1)}_{f_y}\,dy \quad , = f(b) - f(a)$$

$= 1 + \cancel{c} - \cancel{c}$

$\boxed{= 1}$

$\int f_x = F \qquad f = x^4y^2 + x + g(y)$

$f_y = 2\cancel{x^4}y + g'(y) = 2x^4 y + 1$

$g'(y) = 1 \quad g(y) = \int g'(y)$

$= y + C \quad \boxed{\therefore f = x^4 y^2 + x + C}$

over the path

$$\mathbf{r}(t) = \langle \sin t^2, \cos t^2 \rangle \quad , \quad 0 \le t \le \sqrt{\pi/2} \quad .$$

$= \sin^4(t^2)\cos^2(t^2) + \sin(t^2) + C$

Explain!

$f(0) = C \qquad f(\sqrt{\pi/2}) = 1 + C$

:

Exam Problem 2. (10 points) By changing the order of integration, if necessary, evaluate the double-integral

$$\int_0^5 \int_{(y/5)^{1/3}}^1 \sin x^4 \, dx \, dy$$

Here is what I did wrong (if applicable):

Done

Problem 2a:

Change the order of integration

Done in the end

$$\int_0^1 \int_0^{e^x} f(x,y) dy \, dx$$

Problem 2b:

Change the order of integration

$$\int_0^\pi \int_0^{\sin x} f(x,y) dy \, dx$$

Problem 2c:

Change the order of integration

$$\int_0^1 \int_{e^y}^e f(x,y) dx \, dy$$

Done

Exam Problem 3. (10 points) Find the equation of the tangent plane at the point $(1,1,1)$ to the surface given parametrically by

$$x(u,v) = u^3 v \quad , \quad y(u,x) = uv \quad , \quad z(u,v) = uv^3 \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

Done in the end

Here is what I did wrong (if applicable):

Problem 3a. Find the equation of the tangent plane at the point $(1, 2, 4)$ to the surface given parametrically by

$$x(u, v) = u^2 \quad , \quad y(u, x) = uv \quad , \quad z(u, v) = v^2 \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

Problem 3b. Find the equation of the tangent plane at the point $(-1, -1, 2)$ to the surface given parametrically by

$$x(u, v) = u^3 \quad , \quad y(u, x) = v^3 \quad , \quad z(u, v) = -2uv \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

Done

Exam Problem 4. Let $f(x, y, z) = e^{\cos x^2 + \sin xyz + \cos xz}$, and let

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

$F = \nabla f$

Let C be the curve

$$r(t) = \langle \cos t, t, \sin t \rangle \quad , \quad 0 \leq t \leq 2\pi \quad .$$

Find the value of the line-integral

$$\int_C \mathbf{F}.d\mathbf{r} \quad .$$

$r(0) = \langle 1, 0, 0 \rangle$

$r(2\pi) = \langle 1, 0, 0 \rangle$

$= 0$

Explain! Just giving the answer will give you no credit.

Here is what I did wrong (if applicable):

Problem 4a Let $f(x, y, z) = \sin(x + y^2 + z^3)$, and let

$\sin(3 + 81 +$

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

$F = \nabla f \quad \therefore$ conservative

Let C be the curve

$$r(t) = \langle t, t^2, t^3 \rangle \quad , \quad 0 \le t \le 3 \quad .$$

$r(0) = \langle 0, 0, 0 \rangle$
$r(3) = \langle 3, 9, 27 \rangle$

Find the value of the line-integral

$$\int_C \mathbf{F}.d\mathbf{r} \quad .$$

$= \sin(19695)$

Explain! Just giving the answer will give you no credit.

Problem 4b Let $f(x, y) = e^{cosx+3\sin y}$, and let

$$\mathbf{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$$

$r(0) = \langle 0, 1 \rangle$
$r(\pi) = \langle 0, 1 \rangle$

Let C be the curve

$$r(t) = \langle \sin 2t, \cos t \rangle \quad , \quad 0 \le t \le \pi \quad .$$

$= 0$

Find the value of the line-integral

$$\int_C \mathbf{F}.d\mathbf{r} \quad .$$

Explain! Just giving the answer will give you no credit.

Done

Exam Problem 5. (10 points) Evaluate the triple integral

$$\int_R (x^2 + y^2 + z^2)^3 \, dx \, dy \, dz \quad ,$$

Spherical coordinates.

where R is the region in 3D space given by

$\iiint \rho^6 (\rho^2 \sin\phi) \, d\rho \, d\phi \, d\theta$

$\rho^8 \sin\phi$ $\quad \rho = 0..1$

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \le 1 \quad , \quad x, y, z \ge 0\} \quad .$$

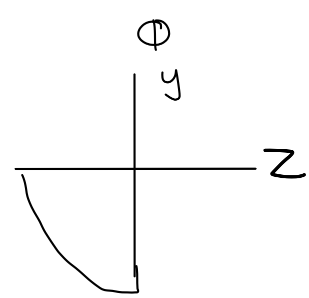
$\phi = 0..\pi/2$

$\theta = 0..\pi/2$

Here is what I did wrong (if applicable):

Problem 5a Evaluate the triple integral

$$\int_R (x + y)(x^2 + y^2 + z^2)^2 \, dx \, dy \, dz \quad ,$$



where R is the region in 3D space given by

$\rho^4 \cdot \rho \sin\phi (\cos\theta + \sin\theta) \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

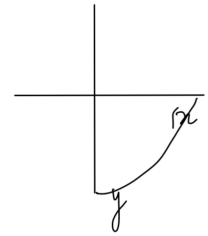
$= \rho^7 \sin^2\phi(\sin\theta+\cos\theta)$

$\rho = 0..1$

$\phi - \pi/2..\pi$

$\theta = 0..\frac{3\pi}{2}$

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \le 1 \quad , \quad x \ge 0, y < 0, z < 0\} \quad .$$



Problem 5b Evaluate the triple integral

$\rho = 0..1$

$\phi = \pi/2..\pi$

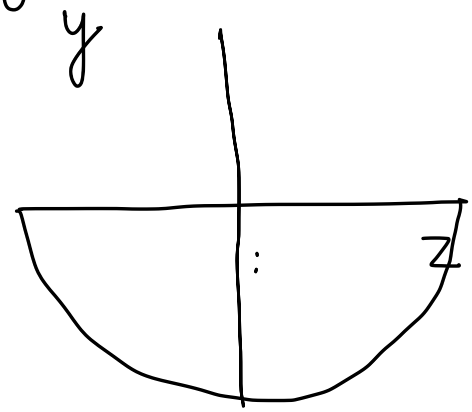
$$\int_R z(x^2 + y^2 + z^2)\, dx\, dy\, dz \quad ,$$

$\int\int\int \rho^4 \cos\phi \sin\phi \, d\rho\, d\phi\, d\theta$

where R is the region in 3D space given by

$\theta = \pi..2\pi$

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \le 1 \quad , \quad y < 0\} \quad .$$



Problem 5c Evaluate the triple integral

$\rho = 0..2\sqrt{2}$

$\phi = 0..\pi$

$\theta = 0..2\pi$

$$\int_R (z - x)\, dx\, dy\, dz \quad ,$$

where R is the region in 3D space given by

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \le 8\} \quad .$$

Done

Exam Problem 6. Evaluate the double integral

$$\int_{-3}^{0} \int_{0}^{\sqrt{9-x^2}} (x^2 + y^2)^2 \, dy\, dx$$

$$\frac{x^2 + y = x^2}{\sqrt{9^2 - x^2}}$$

Here is what I did wrong (if applicable):

Problem 6a Convert the integral to polar coordinates, do not evaluate.

$x=$

$$\int_{-3}^{0} \int_{0}^{\sqrt{9-x^2}} (x^2 + y)\, dy\, dx$$

$\int_{\pi/2}^{\pi} \int_0^3 9\cos\theta + \sin\theta$

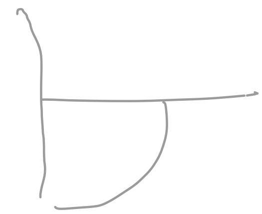
$y^2 = 9 - x^2$

$r = 3$

Problem 6b Convert the integral to polar coordinates, do not evaluate.

$\int_{-\pi/2}^{0}\int_{0}^{4}$

$$\int_0^4 \int_{-\sqrt{16-x^2}}^{0} (x^2+y)\,dy\,dx$$



Problem 6c Convert the integral to polar coordinates, do not evaluate.

$$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{x}^{\sqrt{1-x^2}} (x^3+y^2)\,dy\,dx$$

Don't know

Exam Problem 7. (10 points altogether) Decide whether the following limits exist. If it does find them. If it does not **Explain** why not?

(a) (2 *points*) $\lim\limits_{(x,y)\to(\pi/2,\pi/2)} \dfrac{\cos x + \sin x}{x+y}$, (b) (2 *points*) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x-y}$,

(c) (2 *points*) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-y}{x^2-y^2}$, (d) (4 *points*) $\lim\limits_{(x,y)\to(1,1)} \dfrac{x+y-2}{2x+y-3}$,

Here is what I did wrong (if applicable):

Done

Problem 7a: Decide whether the following limit exists. If it does, find it, if not, explain!

$$\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3} \quad ,$$

$y = 3 + m(x-1)$

$\lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{\cancel{3}+m\cancel{(x-1)}-\cancel{3}}$ $\dfrac{1}{m}$

DNE

Problem 7b: Decide whether the following limit exists. If it does, find it, if not, explain!

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z}$$

$\lim\limits_{x\to 0} \dfrac{x + cx + 2mx}{2x + cx + mx} = \dfrac{1+c+2m}{2+c+m}$

DNE

Exam Problem 8. Compute the line integral $\int_C f\,ds$ where

$$f(x,y,z) = xyz$$

$x = t$

and C is the line segment starting at $(0,0,0)$ and ending at $(1,2,-3)$

$y = 2t$

$z = -3t$

$t = 0$ $t = 1$

$r(t)$

parametrise

$x = t$
$y = t$
$z = -t$

Here is what I did wrong (if applicable):

$r'(t) = \hat{i} + \hat{j} - \hat{k}$
$= \sqrt{3}$

Problem 8a Compute the line integral $\int_C f\, ds$ where

$$f(x, y, z) = xy^2 + yz^2 + z$$

$f(t) = \int (2t^3 - t)\sqrt{3}$

and C is the line segment starting at $(0, 0, 0)$ and ending at $(1, 1, -1)$

Problem 8b Compute the line integral $\int_C f\, ds$ where

$$f(x, y) = x + y$$

and C is the upper circle $\{(x, y) : x^2 + y^2 = 1, y > 0\}$.

Done

Exam Problem 9. Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle z, z, x \rangle \quad ,$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x \geq 0, y \geq 0, z \geq 0$$

with **downward pointing** normal.

Here is what I did wrong (if applicable):

Problem 9a Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle x + z, y + z, -x \rangle \quad ,$$

$\iint \left(-P \frac{dg}{dx} - Q \frac{dg}{dy} + R\right) dA$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x < 0, y < 0, z \geq 0$$

$2x(x+z) + (y+z)(2y) - x$

with **upward pointing** normal.

$\int_{-3}^{0} \int_{-\sqrt{9-x^2}}^{0} [2x(9-x^2-y^2+x) + (y+9-x^2-y^2+y)(2y) - x]\, dy\, dx$

Problem 9b Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle x + z, y + z, -x \rangle \quad ,$$

Same parameter
but different
limit

$$= -\int_0^1 \int_0^1 \quad dy\,dx$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad ,0 < x < 1, 0 < y < 1, z \geq 0$$

with **downward pointing** normal.

Exam Problem 10. Find the **point** on the plane $x + 2y + 3z = 18$ where the function $f(x, y, z) = xyz$ is **as large as possible**.

Here is what I did wrong (if applicable):

Problem 10a Find the **maximum value** of the function $f(x, y, z) = xyz$ on the plane $2x + y + z = 4$

Problem 10b Find the point on the plane $2x + y + z = 4$ where $f(x, y, z) = xy^2z$ is as large as possible. (You can use Maple)

$\nabla f = \langle yz, xz, xy \rangle$ $\qquad \nabla g \langle 2, 1, 1 \rangle$

$yz = \lambda 2 \qquad xz = \lambda \qquad xy = \lambda$

$\frac{\lambda^2}{x^2} = 2\lambda \qquad z = \frac{\lambda}{x} \qquad y = \frac{\lambda}{x}$

$\lambda = 2x^2 \qquad z = 2x \qquad y = 2x$

$x = \sqrt{\frac{\lambda}{2}} \qquad z = 2\sqrt{\frac{\lambda}{2}} \qquad y = 2\sqrt{\frac{\lambda}{2}}$

$= \pm \frac{2}{3} \qquad = \pm \frac{4}{3} \qquad = \pm \frac{4}{3}$

$$2\sqrt{\frac{\lambda}{2}} + 2\sqrt{\frac{\lambda}{2}} + 2\sqrt{\frac{\lambda}{2}} = 4$$

$$3\sqrt{\frac{\lambda}{2}} = 2$$

$$\frac{\lambda}{2} = \frac{4}{9}$$

$$\lambda = \pm\frac{8}{9}$$

$$\boxed{f(x,yz) = \frac{32}{27}}$$

SCC2

Exam Problem 1. (10 pts.) Compute the line integral

$$\int_C \underset{f_x}{yz\,dx} + \underset{f_y}{(xz+z)\,dy} + \underset{f_z}{(xy+y+1)\,dz} \quad ,$$

over the path

$$\mathbf{r}(t) = \langle e^{t^3}, t^2 e^{t^4}, t e^{t^7} \rangle \quad , \quad 0 \le t \le 1 \quad .$$

Explain!

:

Here is what I did wrong (if applicable):

Problem 1a. Compute the line integral

$$\int_C x e^{xyz}\,dx + y e^{xyz}\,dy + z e^{xyz}\,dz \quad ,$$

over the path

$$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \quad , \quad 0 \le t \le 1 \quad .$$

Explain!

Problem 1

$\mathrm{curl}(F) = 0$

$\therefore \quad f(x,y,z) = \int f_x$

$= \int yz\,dx$

$f(x,y,z) = xyz + g(y,z)$

$f_y = xz + g_y(y,z)$

$$\cancel{xz} + z = \cancel{xz} + g_y(y,z)$$

$$g_y(y,z) = z$$

$$g(y,z) = \int g_y(y,z) = \int z$$

$$= yz + c$$

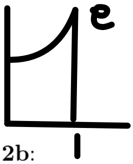
$$g(y,z) = yz + g(z)$$

$$\therefore f(x,y,z) = xyz + yz + g(z)$$

$$f_z = xy + y + g'(z)$$

Problem 2a:

Change the order of integration



$$\int_0^1 \int_0^{e^x} f(x,y)dy\,dx$$

$$\int_0^e \int_{\ln(y)}^1 \qquad dx\,dy$$

Problem 2b:

Change the order of integration

$y = \sin x$

$$\int_0^\pi \int_0^{\sin x} f(x,y)dy\,dx$$

$$\int_0^1 \int_{\sin^{-1}(y)}^3 \quad dx\,dy$$

Problem 2c:

Change the order of integration

$$\int_0^1 \int_{e^y}^e f(x,y)dx\,dy \qquad \int_1^e \int_0^{\ln(x)} dy\,dx$$

Problem 3a. Find the equation of the tangent plane at the point $(1, 2, 4)$ to the surface given parametrically by

$$x(u,v) = u^2 \quad , \quad y(u,x) = uv \quad , \quad z(u,v) = v^2 \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

Problem 3b. Find the equation of the tangent plane at the point $(-1, -1, 2)$ to the surface given parametrically by

N/A

$$x(u,v) = u^3 \quad , \quad y(u,x) = v^3 \quad , \quad z(u,v) = -2uv \quad , \quad -\infty < u < \infty \quad , \quad -\infty < v < \infty \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

(3a)

$1 = u^2$

$u = \pm 1$

$v = 4$

$v = \pm 2$

$2 = uv$

$\therefore u = -1, v = -2$

or

$u = 1, v = 2$

$$r_u = \langle 2u, v, 0 \rangle = \langle 2, 2, 0 \rangle$$

$$r_v = \langle 0, u, 2v \rangle = \langle 0, 1, 4 \rangle$$

$$r_u \times r_v - \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$$8i - 8j + 2k,$$

$$2(z-4) + 8(y-2) + 8(x-1) = 0$$

$$2z + 8y + 8x = 8 + 16 + 8$$

$$z + 4y + 4x = \frac{32}{2} = 16$$

$$z = -4y - 4x + 16$$