second chance club 2.

Q1.(a) $\int_C xe^{xyz}\,dx + ye^{xyz}\,dy + ze^{xyz}\,dz$

$r(t) = (t, t^2, t^3) \quad 0 \le t \le 1$

Ans: $\text{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ xe^{xyz} & ye^{xyz} & ze^{xyz} \end{vmatrix}$

$= (xz^2e^{xyz} - xy^2e^{xyz})i - (yz^2e^{xyz} - x^2ye^{xyz})j + (xy^2ze^{xyz} - x^2ze^{xyz})k$

$\text{grad}(f) = (\cancel{xe^{xyz}}, ye^{xyz}, ze^{xyz})$

$\therefore f_x = xe^{xyz}$

$F = e^{x}$

$x = t \quad y = t^2 \quad z = t^3$

$\int_C t \cdot e^{t^6}\,dt + t^2 \cdot e^{t^6} 2t\,dt + t^3 \cdot e^{t^6} 3t^2\,dt$

$= \int_0^1 e^{t^6}(t + 2t^3 + 3t^5)\,dt$

$= \frac{1}{6t^4}e^{t^6} + \frac{1}{3t^2}e^{t^6} + \frac{1}{2}e^{t^6} \Big|_0^1$

$= e\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{2}\right) - 1\cdot(0)$

$= e$

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Q1 (b). $\int_C (4x^3y^2+1)\,dx + (2x^4y+1)\,dy$

$r(t) = (\sin t^2, \cos t^2)$ $\quad 0 \le t \le \sqrt{\frac{\pi}{2}}$

Ans: $\text{curl}(F) = \begin{vmatrix} \frac{d}{dx} & \frac{d}{dy} \\ 4x^3y^2+1 & 2x^4y+1 \end{vmatrix} = 8x^3y - 8x^3y = (0,0)$

$\therefore$ it is conservative.

$f_x = 4x^3y^2+1$

$f = x^4y^2 + x + g(y)$

$f \cdot \frac{d}{dy} = 2x^4y + g'(y)$

$g'(y) = 1$

$g(y) = y$

$\therefore f = x^4y^2 + x + y$

$r = (\sin 0, \cos 0) = (0,1)$

$r = (\sin\frac{\pi}{2}, \cos\frac{\pi}{2}) = (1,0)$

$\int_C F\,dc = f(1,0) - f(0,1) = 1 - 1 = 0.$

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Q2(a) = $\int_0^1 \int_0^{e^x} f(x,y)\,dy\,dx$

Ans: $\Rightarrow \{0 \le x \le 1 \; , \; 0 \le y \le e^x\}$

$y = e^x$

$\ln y = x$

$\{\ln y \le x \le 1, \; 0 \le y \le e\}$

$\int_0^e \int_{\ln y}^1 f(x,y)\,dx\,dy$

Q2(b) $\int_0^\pi \int_0^{\sin x} f(x,y)\,dy\,dx$

Ans: $\{(x,y) \mid 0 \le x \le \pi \quad 0 \le y \le \sin x\}$.

$y = \sin x$

$x = \sin^{-1} y$

$\therefore \{(x,y) \mid \sin^{-1} y \le x \le \pi \; , 0 \le y \le \pi\}$.

Q2.(c): $\int_0^1 \int_{e^y}^e f(x,y)\,dx\,dy$

$= \{(x,y) \mid 0 \le y \le 1 \quad e^y \le x \le e\}$.

Ans: $e^y = x$ $\{(x,y) \mid \ln x \le y \le 1, \; 0 \le x \le e\}$.

$y = \ln x$

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Q3.(a).

$u^2 = 1 \quad uv = 2 \quad v^2 = 4$

$\therefore (u,v) = (1,2)$ or $(u,v) = (-1,-2)$

$\cancel{f_x =}$ $r(u,v) = \cancel{u^2v + uv + uv}$

$u^2 + uv + v^2$

$r_u = 2u + v = (2u, v, 0)$

$r_v = (0, u, 2v)$

$(1,2)$

$r_u = (2,2,0)$

$r_v = (0,1,4)$

$N = \begin{vmatrix} 2, 2, 0 \\ 0, 1, 4 \end{vmatrix}$

$= 8i - 8j + 2k$

$= (8, -8, 2)$

$(-1,-2)$

$r_u = (-2,-2,0)$

$r_v = (0,-1,-4)$

$N = \begin{vmatrix} -2, -2, 0 \\ 0, -1, -4 \end{vmatrix}$

$= 8i - 8j + 2k$

$= (8, -8, 2)$

$\cancel{\therefore N.}$

$z-4 = 8(x-1) + (-8)(y-2) + \cancel{z(z-4)}$

$z = 8x - 8y + 12$

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Q4.(a) because $F = (\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz})$

$\therefore$ F is conservative vector field.

$r(0) = (0,0,0)$ $\quad r(3) = (3,9,27)$

$\therefore \int_C F \cdot dr = f(3,9,27) - f(0,0,0)$

$= \sin(3+81+27^2) - \sin 0$

$= \sin 813$

≈ 0.67294

Q4(b). because $F = (\frac{df}{dx}, \frac{df}{dy})$

$\therefore$ F is conservative vector field

$r(0) = (0,1)$ $\quad r(\pi) = (0,-1)$

$\int_C F \cdot dr = f(0,-1) - f(0,1)$

$= e^{1+3\sin -1} - e^{\cos 0 + 3\sin 1}$

$= -33.7160$

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Q5(a) $\int_R (x+y)(x^2+y^2+z^2)^2 \, dx\,dy\,dz$

$\{(x,y,z) \mid x^2+y^2+z^2 \leq 1 \quad x \geq 0 \; y < 0, \; z < 0\}$.

Ans: $x^2+y^2+z^2 = \rho^2$ $\quad \pi \leq \theta \leq \frac{3}{2}\pi$.

$\rho^2 \leq 1$ $\cancel{\frac{3}{2}\pi \leq \theta \leq 2\pi \quad 0 \leq \phi \leq}$ $\frac{3}{2}\pi \leq \phi \leq 2\pi\}$.

$$\int_{\pi}^{\frac{3}{2}\pi} \int_{\frac{3}{2}\pi}^{2\pi} \int_0^1 \underset{(\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta)}{\cancel{(r\cos\theta + r\sin\theta)}} \cdot \rho^4 \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\Rightarrow \int_0^1 \rho^7 (\sin\phi\cos\theta + \sin\phi\sin\theta) \overset{\sin\theta}{} d\rho$$

$$= \frac{1}{8}\rho^8 \Big|_0^1 (\sin\phi\cos\theta + \sin\phi\sin\theta) \cdot \sin\theta$$

$$= \int_{\pi}^{\frac{3}{2}\pi} \int_{\frac{3}{2}\pi}^{2\pi} \frac{1}{8}(\sin\phi\cos\theta + \sin\phi\sin\theta) \cdot \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \int_{\frac{3}{2}\pi}^{2\pi} \frac{\sin\phi}{8} \cdot (\cos\theta + \sin\theta) \cdot \sin\theta \, d\theta$$

$$= -\frac{1}{2} + \frac{\pi}{4}$$

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Q5(b) $\rho^2 \le 1$ $\rho = 0..1$ $y < 0$

$\therefore \theta = \frac{\pi}{2} .. \frac{3}{2}\pi$ $\phi = \frac{\pi}{2} .. \frac{3}{2}\pi$.

$$\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \int_0^1 \rho\cos\phi \cdot (\rho^2) \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \frac{\pi\left(-\frac{\sin(\frac{\pi}{2})^2}{2} + \frac{\sin(\frac{3\pi}{2})^2}{2}\right)}{6} \approx -0.2194$$

Q5(c). $\rho^2 \le 8$ $\rho = 0..\sqrt{8}$ $\phi = 0..2\pi$ $\theta = 0..2\pi$

$$\int_0^{2\pi} \int_0^{2\pi} \int_0^{2\sqrt{2}} \rho(\cos\phi - \sin\phi\cos\theta)\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \int_0^{2\pi} \frac{\rho^4}{4}\Big|_0^{2\sqrt{2}} (\cos\phi - \sin\phi\cos\theta) \cdot \sin\phi \, d\theta \, d\phi.$$

$$= 2\int_0^{2\pi} \int_0^{2\pi} \cos\phi\sin\phi - \sin\phi^2\cos\theta \, d\theta \, d\phi$$

$$= 2\int_0^{2\pi} -\sin\phi^2 \sin\theta + \cos\phi\sin\phi \cdot \theta \Big|_0^{2\pi}.$$

$$= 2\int_0^{2\pi} \cos\phi\sin\phi \cdot 2\pi \, d\phi$$

$$= 4\pi \frac{\sin(2\phi)^2}{2}\Big|_0^{2\pi}$$

$$= 0.$$

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Q6. (a) $r = 0..3$ $\theta = \frac{\pi}{2} .. \pi$

$$\int_{\frac{\pi}{2}}^{\pi}\int_0^3 r((r\cos\theta)^2 + r\sin\theta)\,dr\,d\theta$$

$$= \int_0^3 r \cdot r^2 \cos\theta^2 + r^2 \sin\theta \, dr$$

$$= \frac{r^4}{4}\cos\theta^2 + \frac{r^3}{3}\sin\theta \Big|_0^3$$

$$= 20.25\cos\theta^2 + 9\sin\theta$$

$$\int_{\frac{\pi}{2}}^{\pi} 20.25\cos\theta^2 + 9\sin\theta\,d\theta$$

$$= \frac{81}{16}\pi + 9$$

Q6 (b). $\int_0^4 \int_{-\sqrt{16-x^2}}^{0} (x^2+y)\,dy\,dx$

$r = 0..4$ $\theta = \pi .. 2\pi$

$$\int_{\pi}^{2\pi}\int_0^4 ((r\cos\theta)^2 + r\sin\theta)\,r\,dr\,d\theta$$

$$= 32\pi - \frac{64}{3}\cdot 2$$

Q6 (c). $r = \frac{\sqrt{2}}{2}$ $\theta = 0..\pi$

$$\int_0^{\pi}\int_0^{\frac{\sqrt{2}}{2}} \left((r\cos\theta)^3 + (r\sin\theta)^2\right) r\,dr\,d\theta$$

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Q8 ~~(a)~~: the problem I made in exam is I forget to multiply the integrand by ~~(t)~~ $|r'(t)|$

Q8(a): $P + t(Q-P)$

$= (t, t, -t)$

$r'(t) = (1, 1, -1)$

$|r'(t)| = \sqrt{3}\,dt$

$\int_0^1 t^3 + t^3 + (-t)\,dt \cdot \sqrt{3}$

$= \int_0^1 (2t^3 + (-t))\sqrt{3}\,dt$

$= \frac{\sqrt{3}}{2} t^4 - \frac{\sqrt{3}}{2} t^2 \Big|_0^1$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0.$

Q8(b). $r = 1 \quad \theta = 0..\pi$

$\int_0^\pi \int_0^1 (r\cos\theta + r\sin\theta) \cdot r \, dr\, d\theta$

$= \int_0^\pi \frac{r^3}{3}(\cos\theta + \sin\theta)\Big|_0^1 d\theta$

$= \frac{1}{3} \int_0^\pi (\cos\theta + \sin\theta)\, d\theta$

$= \frac{1}{3}(\sin\theta - \cos\theta)\Big|_0^\pi$

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

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Q9(a). $\frac{dg}{dx} = -2x, \quad \frac{dg}{dy} = -2y.$

$P = x+z \quad Q = y+z \quad R = -x$

$$\int_{-3}^{0}\int_{-\sqrt{9-x^2}}^{0} \left(+\left(x+(9-x^2-y^2)\right)\cdot 2x + \left(y+(9-x^2-y^2)\right)\cdot 2y + (-x)\right) dy\,dx$$

$$= \int_{-3}^{0}\int_{-\sqrt{9-x^2}}^{0} 2x^2+2y^2-x+(9-x^2-y^2)\cdot(2x+2y)\,dy\,dx$$

$$= \frac{81}{8}\pi - \frac{648}{5}$$

Q9(b).

$$\int_0^1\int_0^1 \left(\left(x+9-x^2-y^2\right)\cdot 2x + \left(y+9-x^2-y^2\right)\cdot 2y - x\right) dy\,dx$$

$= 17.$

$\because$ downward

$\therefore$ it is -17.

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Q10.(a) $(yz, xz, xy) = (2,1,1)\lambda$

$yz = 2\lambda$ $\quad xz = \lambda$ $\quad xy = \lambda$ $\quad y = 2x$ $\quad z = y$

$\cancel{xz = xy}$ $\quad \cancel{y^2 = 2\lambda}$ $\quad \frac{y}{x} = 2$ $\quad \frac{z}{y} = 1$

$z = y$ $\quad \cancel{xy = \lambda}$ $\quad \frac{z}{x} = 2$

$z\cancel{x} = y,$

$2x + 2x + 2x = 4$ $\quad y = \frac{4}{3}$ $\quad z = \frac{4}{3}$

$6x = 4$

$x = \frac{2}{3}$

$\therefore \left(\frac{2}{3}, \frac{4}{3}, \frac{4}{3}\right) \Rightarrow f(x,y,z) = \frac{32}{27}$

Q10(b). $(y^2z, 2xyz, xy^2) = (2,1,1)\lambda$

$y^2z = 2\lambda$ $\quad \frac{z}{x} = 2$ $\quad \frac{y}{2x} = 2$

$2xyz = 1\lambda$ $\quad z = 2x$ $\quad y = 4x$

$xy^2 = 1\lambda$

$2x + 4x + 2x = 4$ $\quad y = 2$ $\quad z = 1$

$8x = 4$

$x = \frac{1}{2}$

the point is. $\left(\frac{1}{2}, 2, 1\right)$