



Exam problem 1: compute the line integral.

$$\int_C yz dx + (xz + z) dy + (xy + y + 1) dz$$

over the path.

$$r(t) = \langle e^t, t^2 e^{t^4}, t e^{t^7} \rangle, 0 \leq t \leq 1$$

Explain!

Ans: $dx = t^2 e^{t^2-1} dt$ $dy = t^6 e^{t^4-1} dt$ $dz = t^8 e^{t^7-1} dt$

$$\int_0^1 t^2 e^{t^2} \cdot t e^{t^7} \cdot t^2 e^{t^2-1} dt + (e^{t^2} \cdot t e^{t^7} + t e^{t^7}) \cdot t^6 e^{t^4-1} dt + (e^{t^2} \cdot t^2 e^{t^4} + t^2 e^{t^4} + 1) \cdot t^8 e^{t^7-1} dt$$

$$= \int_0^1 t^6 e^{t^2+t^7+t^2-1} + (t e^{t^2+t^7} + t e^{t^7}) \cdot t^6 e^{t^4-1} + (t^2 e^{t^2+t^4} + t e^{t^4} + 1) \cdot t^8 e^{t^7-1} dt$$

$$= \int_0^1 t^6 e^{t^4+t^7+t^2-1} + t^7 e^{t^2+t^7+t^4-1} + t^7 e^{t^7+t^7-1} + t^{10} e^{t^2+t^4+t^7-1} + t^9 e^{t^7-1} dt$$

$$= (1 + e^2 + e) e$$

Ans: $(1 + e^2 + e) e$

problem 1a. Compute the line integral

$$\int_C x e^{xy^2} dx + y e^{xy^2} dy + z e^{xy^2} dz$$

over the path.

$$r(t) = \langle t, t^2, t^3 \rangle, 0 \leq t \leq 1$$

explain!

Ans: $dx = dt$ $dy = 2t dt$ $dz = 3t^2 dt$
 $\int_0^1 t \cdot e^{t^3} dt + t^2 e^{t^3} \cdot 2t dt + t^3 e^{t^3} \cdot 3t^2 dt = \int_0^1 e^{t^3} (t + 2t^3 + 3t^5) dt \approx 2.31$

7. problem 1b compute the line integral

$$\int_C (x^2y^2+1)dx + (x^2y+1)dy,$$

over the path.

$$r(t) = \langle \sin t^2, \cos t^2 \rangle, \quad 0 \leq t \leq \sqrt{\frac{\pi}{2}}$$

Explain!

$$dx = 2t \cos t^2 dt \quad dy = -2t \sin t^2 dt$$

$$\int_0^{\sqrt{\frac{\pi}{2}}} (4 \sin^2 t^2 \cdot \cos^2 t^2 + 1) \cdot (2t \cos t^2) dt + (2 \sin^2 t^2 \cos t^2 + 1) \cdot (-2t \sin t^2) dt$$

≈ 2.59

Ans: 2.59

example 2: By changing the order of integration, if necessary, evaluate the double-integral.

$$\int_0^{\frac{1}{2}} \int_{\frac{1}{6}}^{\frac{1}{2}} \sin x^4 dx dy$$

Here is what I did wrong (if applicable):

$$\begin{aligned} \text{Ans: } & \int_0^{\frac{1}{2}} \int_0^{\frac{1}{6}} \sin x^4 dy dx \\ &= \int_0^{\frac{1}{2}} \sin x^4 dx \\ &= \frac{1}{4} \int_0^{\frac{1}{2}} \sin u du \\ &= \frac{1}{4} [-\cos u] \\ &= \frac{1}{4} (-\cos \frac{1}{4} + \cos 0) = -\frac{1}{4} \cos \frac{1}{4} + \frac{1}{4} \end{aligned}$$

Ans: $-\frac{1}{4} \cos 1 + \frac{1}{4}$

7. Problem 2a:

changing the order of integration

$$\int_0^1 \int_0^{e^x} f(x, y) dy dx$$

Ans: $\int_0^e \int_{\ln y}^1 f(x, y) dx dy$

7. Problem 2b:

changing the order of integration

$$\int_0^{\pi} \int_0^{\sin x} f(x, y) dy dx$$

Ans: $\int_0^1 \int_{\arcsin y}^{\pi} f(x, y) dx dy$

7. Problem 2c:

changing the order of integration

$$\int_0^1 \int_0^e f(x, y) dx dy$$

Ans: $\int_0^e \int_0^{\min(x, 1)} f(x, y) dy dx$

CORE i7

GEFORCE
RTX

exam problem? Find the equation of the tangent plane at the point $(1, 1, 1)$ to the surface given parametrically by

$$x(u, v) = u^3v, \quad y(u, v) = uv, \quad z(u, v) = uv^3,$$

$$-\infty < u < \infty, \quad -\infty < v < \infty.$$

Express your answer in explicit form, i.e. in the format $z = ax + by + c$.

$$\# \text{Ans: } f_u = \langle 3u^2v, v, v^3 \rangle$$

$$\# \text{ } f_v = \langle u^3, u, 3uv^2 \rangle$$

$$u^3v = 1 \quad uv = 1 \quad uv^3 = 1$$

$$u = \pm 1 \quad v = \pm 1.$$

$$\textcircled{1} u = 1 \quad v = 1$$

$$f_u = \langle 3, 1, 1 \rangle \quad f_v = \langle 1, 1, 3 \rangle$$

$$f_u \times f_v = \langle 2, -8, 2 \rangle = \langle 1, -4, 1 \rangle$$

$$\textcircled{2} u = -1 \quad v = -1$$

$$f_u = \langle -3, -1, -1 \rangle \quad f_v = \langle -1, -1, -3 \rangle$$

$$f_u \times f_v = \langle 1, -4, 1 \rangle$$

$$(x-1) - 4(y-1) + (z-1) = 0$$

$$x-1 - 4y + 4 + z-1 = 0$$

$$\boxed{z = -x + 4y - 2}$$

$$\text{Ans: } z = -x + 4y - 2.$$

problem 3a:

Find the equation of the tangent plane at the point $(1, 2, 4)$ to the surface given parametrically by

$$x(u, v) = u^2, \quad y(u, v) = uv, \quad z(u, v) = v^2, \\ -\infty < u < \infty, \quad -\infty < v < \infty.$$

express your answer in explicit form,
i.e. in the format $z = ax + by + c$.

$$u^2 = 1 \quad uv = 2 \quad v^2 = 4$$

$$u = \pm 1 \quad v = \pm 2$$

~~① $u = 1$~~

$$f_u = \langle 2u, v, 0 \rangle$$

$$f_v = \langle 0, u, 2v \rangle$$

① $u = 1 \quad v = 2$

$$f_u = \langle 2, 2, 0 \rangle \quad f_v = \langle 0, 1, 4 \rangle$$

$$\nabla u \times \nabla v = \langle 8, 8, 2 \rangle = \langle 4, 4, 1 \rangle$$

② $u = -1 \quad v = -2$

$$f_u = \langle -2, -2, 0 \rangle \quad f_v = \langle 0, -1, -4 \rangle$$

$$\nabla u \times \nabla v = \langle 4, 4, 1 \rangle$$

$$4(x-1) + 4(y-2) + (z-4) = 0$$

$$4x - 4 + 4y - 8 + z - 4 = 0$$

$$z = -4x - 4y + 16$$

Ans: $z = -4x - 4y + 16$

7. Problem 3b. Find the equation of the tangent plane at the point $(-1, -1, 2)$ to the surface Σ given parametrically by

$$x(u, v) = u^3, \quad y(u, v) = v^3, \quad z(u, v) = -2uv,$$

$$-\infty < u < \infty, \quad -\infty < v < \infty.$$

Express your answer in explicit form, i.e. in the format $z = ax + by + c$.

$$\text{Ans: } u^3 = -1 \quad v^3 = -1 \quad -2uv = 2.$$

Exam problem 4.

Let $f(x, y, z) = e^{\cos x^2 + \sin x y z + \cos x z}$ and let

$$F = \left\langle \frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz} \right\rangle$$

Let C be the curve

$$r(t) = \langle \cos t, t, \sin t \rangle \quad 0 \leq t \leq 2\pi.$$

Find the value of the line-integral

$$\int_C F \cdot dr$$

Explains! just giving the answer will give you no credit.

Ans: $F = \text{grad } f$

$$\text{curl}(\text{grad } f) = \langle 0, 0, 0 \rangle$$

conservative

$$\therefore \int_C F \cdot dr = f(\text{end}) - f(\text{start})$$

$$r(0) = \langle 1, 0, 0 \rangle$$

$$r(2\pi) = \langle 1, 2\pi, 0 \rangle$$

$$\int_C F \cdot dr = e^{\cos 1 + \sin 0 + \cos 0} - e^{\cos 1 + \sin 0 + \cos 0} = 0.$$

Ans: 0

problem 4a

Let $f(x, y, z) = \sin(x + y^2 + z^3)$, and let

$$F = \left\langle \frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz} \right\rangle$$

Let C be the curve

$$r(t) = \langle t, t^2, t^3 \rangle, \quad 0 \leq t \leq 3$$

Find the value of the line integral.

$$\int_C F \cdot dr$$

Explain! just giving the answer will give you no credit.

Ans: $F = \text{grad} f$
 $\text{curl}(\text{grad} f) = \langle 0, 0, 0 \rangle$
conservative.

$$\int_C F \cdot dr = f(\text{end}) - f(\text{start})$$

$$r(0) = \langle 0, 0, 0 \rangle$$

$$r(3) = \langle 3, 9, 27 \rangle$$

$$\begin{aligned} \int_C F \cdot dr &= \sin(3 + 81 + 27^3) - \sin(0) \\ &= \sin(19767) \end{aligned}$$

$$\text{Ans: } \sin(19767)$$

problem 4b Let $f(x,y) = e^{\cos x + 3\sin y}$ and let

$$F = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

Let c be the curve

$$r(t) = \langle \sin 2t, \cos t \rangle \quad 0 \leq t \leq \pi$$

Find the value of the line-integral

$$\int_c F dr$$

Explain! just giving the answer will give you no credit.

Ans: $F = \text{grad } f$
 $\text{curl}(\text{grad } f) = \langle 0, 0 \rangle$
conservative.

$$\int_c F dr = f(\text{end}) - f(\text{start})$$

$$r(0) = \langle 0, 1 \rangle$$

$$r(\pi) = \langle 0, -1 \rangle$$

$$\int_c F dr = e^{\cos 0 + 3\sin \pi} - e^{\cos \pi + 3\sin \pi}$$

$$= e^{1+3\sin \pi} - e^{-1+3\sin \pi}$$

Problem 5 exam problem 5 Evaluate the triple integral

$$\int_R (x^2 + y^2 + z^2)^3 dx dy dz$$

where R is the region in 3D space given by

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1, x, y, z \geq 0\}$$

Ans: $\{(r, \theta, \phi) \mid r=0..1, \theta=0.. \frac{\pi}{2}, \phi=0.. \frac{\pi}{2}\}$

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 (r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta)^3 r^2 \sin \theta dr d\theta d\phi$$

$$= \frac{\pi}{16}$$

Ans: $\frac{\pi}{16}$

Problem 5a evaluate the triple integral

$$\int_R (x+y)(x^2 + y^2 + z^2)^2 dx dy dz$$

where R is the region in 3D space given by

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1, x \geq 0, y < 0, z < 0\}$$

Ans: $\{(r, \theta, \phi) \mid r=0..1, \theta= \frac{3\pi}{2}.. 2\pi, \phi= \frac{\pi}{2}.. \pi\}$

$$\int_{\frac{\pi}{2}}^{\pi} \int_{\frac{3\pi}{2}}^{2\pi} \int_0^1 (r \sin \theta \cos \phi + r \sin \theta \sin \phi) (r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta)^2 r^2 \sin \theta dr d\theta d\phi$$

$$= 0$$

Problem 5b evaluate the triple integral

$$\int_R z(x^2 + y^2 + z^2) dx dy dz$$

where R is the region in 3D space given by

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1, y \geq 0\}$$

Ans:

$$\{(r, \theta, \phi) \mid r=0..1, \theta=0..2\pi, \phi=0..\pi\}$$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^1 r \cos \phi (r^2 \sin^2 \phi \cos^2 \theta + r^2 \sin^2 \phi \sin^2 \theta + r^2 \cos^2 \phi) r^2 \sin \phi dr d\theta d\phi$$

$$= \frac{32\pi}{6}$$

problem 5c evaluate the triple integral

$$\int_R (z - x) dx dy dz$$

where R is the region in 3D space given by

$$\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 8\}$$

$$\{(r, \theta, \phi) \mid r=0..\sqrt{8}, \theta=0..2\pi, \phi=0..\pi\}$$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^{\sqrt{8}} (r \cos \phi - r \sin \phi \cos \theta) \cdot r^2 \sin \phi dr d\theta d\phi$$

$$= 32\pi$$

exam problem 6 Evaluate the double integral

$$\int_{-3}^0 \int_0^{\sqrt{4-x^2}} (x^2 + y^2)^2 dy dx$$

$$\int_{-3}^0 \int_0^{\sqrt{4-x^2}}$$

$$\{(r, \theta) \mid r=0..3 \quad \theta = \frac{\pi}{2}.. \pi\}$$

$$\begin{aligned} & \int_{\frac{\pi}{2}}^{\pi} \int_0^3 r^2 \cdot r dr d\theta & \int_{\frac{\pi}{2}}^{\pi} \int_0^3 r^5 dr d\theta \\ & = \int_{\frac{\pi}{2}}^{\pi} \int_0^3 r^3 dr d\theta & = \frac{243}{4} \pi \\ & = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{4} r^4 \Big|_0^3 d\theta \\ & = \frac{81}{4} \cdot \frac{\pi}{2} \\ & = \frac{81}{8} \pi \end{aligned}$$

problem 6a: convert the integral to polar coordinates,
do not evaluate

$$\int_{-3}^0 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx$$

$$\{(r, \theta) \mid r=0..3 \quad \theta = \frac{\pi}{2}.. \pi\}$$

$$\text{Ans: } \int_{\frac{\pi}{2}}^{\pi} \int_0^3 (r \cos \theta)^2 + (r \sin \theta) r dr d\theta$$

Problem 6b: Convert the integral to the polar coordinates
do not evaluate

$$\int_0^4 \int_{-\sqrt{6-x}}^0 (x^2+y) dy dx$$

$$(r, \theta) \quad (r=0..4 \quad \theta = \frac{3}{2}\pi \dots 2\pi)$$

$$\text{Ans: } \int_{\frac{3}{2}\pi}^{2\pi} \int_0^4 (r \cos \theta)^2 + r \sin \theta \cdot r dr d\theta$$

Problem 6c: Convert the integral to polar coordinates
do not evaluate.

$$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_x^{\sqrt{1-x^2}} (x^3+y^2) dy dx$$

$$\text{Ans: } \int_0^{\frac{\sqrt{2}}{2}} \int_0^{\sqrt{1-x^2}}$$

Exam Problem 7 Decide whether the following limits exist. If it does find them. If it does not explain why not?

(a) $\lim_{(x,y) \rightarrow (\frac{\pi}{2}, \frac{\pi}{2})} \frac{\cos x + \sin x}{x+y}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2-y^2}{x-y}$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{x^2-y^2}$

(d) $\lim_{(x,y) \rightarrow (1,1)} \frac{x+y-2}{2x+y-3}$

(a) $\frac{0+1}{1} = 1$ (b) $\frac{(x+y)(x-y)}{x-y} = x+y = 0$

(c) $\frac{x-y}{(x+y)(x-y)} = \text{does not exist.}$ (d) $(x,1) \quad \frac{x-1}{2x-2} = \frac{1}{2}$
 $(1,y) \quad \frac{y-1}{y-1} = 1$ does not exist.

Problem 7a: Decide whether the following limit exists. If it does find it, if not, explain!

$\lim_{(x,y) \rightarrow (1,3)} \frac{x-1}{y-3}$

$(1,3) \frac{x-1}{y-3} \text{ does not exist.}$

$(1,4) \frac{0}{1} = 0$

does not exist.

Problem 7b: Decide whether the following limit exists. If it does find it, if not, explain!

$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x+yz}{2x+yz}$

$A. (x,0,0) \frac{x}{2x} = \frac{1}{2}$

$P. (0,y,0) \frac{y}{y} = 1$

$(0,0,z) \frac{z}{z} = 1$

does not exist.

Exam problem 8: Compute the line integral $\int_C f ds$ where

$f(x,y,z) = xyz$

and C is the line segment starting at $(0,0,0)$ and ending at $(1,2,3)$

$(1,2,3) \quad \text{line: } (t, 2t, 3t) \quad 0 \leq t \leq 1$

$ds = \sqrt{1+4+9} = \sqrt{14}$

$\int_0^1 -6t^3 \sqrt{14} dt = -\frac{3}{2}\sqrt{14}$

ANS: $-\frac{3}{2}\sqrt{14}$

problem 8a: Compute the line integral $\int_C f ds$ where

$$f(x, y, z) = xy^2 + yz^2 + z$$

and C is the line segment starting at $(0, 0, 0)$ and ending at $(1, 1, -1)$.

$$r(t) = \langle t, t, -t \rangle \quad 0 \leq t \leq 1.$$

$$ds = \sqrt{1+1+1} = \sqrt{3} dt$$

$$\begin{aligned} & \int_0^1 (t^3 + t^3 - t) \sqrt{3} dt \\ &= \int_0^1 \sqrt{3} t^3 + \sqrt{3} t^3 - \sqrt{3} t dt \\ &= \frac{\sqrt{3}}{4} t^4 + \frac{\sqrt{3}}{4} t^4 - \frac{\sqrt{3}}{2} t^2 \Big|_0^1 \\ &= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \\ &= 0 \end{aligned}$$

Ans: 0

problem 8b: Compute the line integral $\int_C f ds$ where

$$f(x, y) = x^2 y$$

and C is the upper circle $\{(x, y) : x^2 + y^2 = 1, y > 0\}$.

$$\{(r, \theta) : r = 0..1 \quad \theta = 0.. \pi\}$$

$$\begin{aligned} & \int_0^\pi \int_0^1 (r \cos \theta + r \sin \theta) r dr d\theta \\ &= \int_0^\pi \int_0^1 r^2 \cos \theta + r^2 \sin \theta dr d\theta \\ &= \int_0^\pi \left[\frac{1}{3} r^3 \cos \theta + \frac{1}{3} r^3 \sin \theta \right]_0^1 d\theta \\ &= \int_0^\pi \left[\frac{1}{3} \cos \theta + \frac{1}{3} \sin \theta \right] d\theta = \left[\frac{1}{3} \sin \theta - \frac{1}{3} \cos \theta \right]_0^\pi \\ &= \left(0 + \frac{1}{3} \right) - \left(0 - \frac{1}{3} \right) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \end{aligned}$$

Ans: $\frac{2}{3}$

Exam problem: compute the vector field surface integral $\iint_S \mathbf{F} \cdot d\mathbf{s}$ if $\mathbf{F} = \langle z, z, x \rangle$

$$\mathbf{F} = \langle z, z, x \rangle$$

and S is the oriented surface.

$$z = 4 - x^2 - y^2, \quad x \geq 0, \quad y \geq 0, \quad z \geq 0.$$

with downward pointing normal.

Here is what I did wrong (if applicable): ^{be carefully for region.}

$$\iint_S \mathbf{F} \cdot d\mathbf{s} = \iint_D \left(-P \frac{dy}{dx} - Q \frac{dy}{dy} + R \right) dA.$$

since downward pointing.

$$\iint_S \mathbf{F} \cdot d\mathbf{s} = - \iint_D \left(-P \frac{dy}{dx} - Q \frac{dy}{dy} + R \right) dA.$$

$$\frac{dy}{dx} = -2x \quad \frac{dy}{dy} = -2y$$

$$\iint_S \mathbf{F} \cdot d\mathbf{s} = - \int_0^3 \int_0^{\sqrt{4-x^2}} (zxz + zy z + x) dy dx$$

$$= - \int_0^3 \int_0^{\sqrt{4-x^2}} (zx(4-x^2-y^2) + zy(4-x^2-y^2) + x) dy dx$$

$$= - \frac{643}{5}$$

problem 9a compute the vector-field surface integral $\iint_S F \cdot ds$ of F is

$$F = \langle x+z, y+z, -x \rangle$$

and S is the oriented surface.

$z = 9 - x^2 - y^2$, $x < 0$, $y < 0$, $z \geq 0$.
with upward pointing normal.

$$\text{Ans: } \iint_S F \cdot ds = \iint_D \left(-P \frac{dz}{dx} - Q \frac{dz}{dy} + R \right) dA$$

$$\frac{dz}{dx} = -2x \quad \frac{dz}{dy} = -2y$$

$$\int_{-\sqrt{9-x^2}}^0 \int_{-\sqrt{9-x^2}}^0 (2x^2 + 2xz + 2y^2 + 2yz - x) dy dx$$

$$= -\frac{603}{5} + \frac{81}{4} \pi$$

problem 9b: compute the vector-field surface integral $\iint_S F \cdot ds$ of F is

$$F = \langle x+z, y+z, -x \rangle$$

and S is the oriented surface.

$z = 9 - x^2 - y^2$, $0 < x < 1$, $0 < y < 1$, $z \geq 0$.
with downward pointing normal.

Exam problem 10 Find the point on the plane $x+2y+3z=18$ where the function $f(x,y,z)=xyz$ is as large as possible.

Here is what I did wrong (if applicable):

goal function: $f(x,y,z)=xyz$

constraint function $g = x+2y+3z$

$$\text{grad } f = \lambda \cdot \text{grad } g$$

$$\langle yz, xz, xy \rangle = \lambda \cdot \langle 1, 2, 3 \rangle$$

$$yz = \lambda$$

$$xz = 2\lambda$$

$$xy = 3\lambda$$

$$x+2y+3z=18$$

$$\frac{x}{y} = 2$$

$$y = 2x$$

$$\frac{y}{z} = \frac{3}{2}$$

$$z = \frac{2}{3}y$$

$$x = 2y = 3z$$

$$3x = 18$$

$$x = 6$$

$$y = 3$$

$$z = 2$$

point $\langle 6, 3, 2 \rangle$

value: 36

I know you only ask point.

problem 10a: Find the maximum value of the function
 $f(x, y, z) = xyz$ on the plane $2x + y + z = 4$.

goal function $f = xyz$

constraint function: $g = 2x + y + z$.

$\text{grad } f = \lambda \cdot \text{grad } g$

$$\langle yz, xz, xy \rangle = \lambda \cdot \langle 2, 1, 1 \rangle$$

$$yz = 2\lambda \quad \frac{y}{x} = 2$$

$$xz = \lambda$$

$$xy = \lambda \quad z = y = 2x$$

$$\frac{z}{y} = 1$$

$$z = y$$

$$2x + y + z = 4$$

$$\text{value: } \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} = \frac{64}{27}$$

$$\begin{aligned} 3y &= 4 \\ z &= y = \frac{4}{3} \end{aligned}$$

$$\text{point } \left\langle \frac{2}{3}, \frac{4}{3}, \frac{4}{3} \right\rangle$$

$$x = \frac{2}{3}$$

I know you ask value.

problem 10b Find the point on the plane
 $2x + y + z = 4$ where $f(x, y, z) = xy^2z$ is as
 large as possible (you can use maple).

ANS:

$$\text{grad } f = \lambda \cdot \text{grad } g$$

$$\langle y^2z, 2xy^2z, xy^2z \rangle = \lambda \langle 2, 1, 1 \rangle$$

$$y^2z = 2\lambda$$

$$2xy^2z = 2\lambda$$

$$\frac{y}{x} = \frac{2}{y^2z}$$

$$2xy^2z = \lambda$$

$$2xy^2z = \lambda$$

$$\frac{y}{2x} = 2$$

$$4x^2y = \lambda$$

$$y = 4x$$

$$xy^2z = \lambda$$

$$= 2$$

$$2x + y + z = 4$$

$$\frac{z}{x} = 2$$

$$z = 2x = 1$$

$$\text{point } \left\langle \frac{1}{2}, 2, 1 \right\rangle$$

$$2x + 4x + 2x = 4$$

$$\text{value: } 2$$

$$8x = 4$$

$$x = \frac{1}{2}$$

I know you ask point