Scc1 Rahul Paleja

Problems From A Previous Final:

12.1 → None
12.2 → None
12.3 → None
12.4 → None

12.5 → Find An Equation For The Plane Through the point $(1,0,2)$ that contains the line
$r(t) = \langle 1,1,1\rangle + t\langle 1,-1,0\rangle$
Simplify As Much As You Can!

Answer: $P \to (1,0,2)$ $Q \to (1,1,1)$ Direction Vector $\to \langle 1,-1,0\rangle$

$PQ = \langle 0,1,-1\rangle$

$PQ \times$ Direction Vector $= \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix}$ $\quad i(-1) - j(1) + k(-1)$
$= -i - j - k$
$= \langle -1,-1,-1\rangle$

Using Point $(1,0,2)$
Equation is:
$-(x-1) - (y) - (z-2) = 0$
$-x + 1 - y - z + 2 = 0$
$-x - y - z = -3$ → $\boxed{x+y+z=3}$

Similar Problem:

Find An Equation From The Plane Through The Point $(6,1,0)$ that contains the line $r(t) = \langle 0,1,1\rangle + t\langle 1,1,2\rangle$

Answer: $P \to (6,1,0)$ $Q \to (0,1,1)$ Direction Vector $\to \langle 1,1,2\rangle$

$PQ = \langle -6,0,1\rangle \times$ Direction Vector:

$\begin{vmatrix} i & j & k \\ -6 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix}$

$i(-1) - j(-13) + k(-6) = -i + 13j - 6k$
$\langle -1, 13, -6\rangle$

→ Equation:
$-1(x-6) + 13(y-1) - 6(z) = 0$
$-x + 6 + 13y - 13 - 6z = 0$
$-x + 13y - 6z = 7$
$(-(x - 13y + 6z = 7)) \cdot -1$

Answer → $x - 13y + 6z = -7$

13.1 → None

13.2 → Find the velocity and position vectors of a particle whose acceleration is $a(t) = i + j$ and time $t=0$, the velocity is $i-j$ and position is K

Answer: $\int a(t) = \int i + j \, dt$

$v(t) = ti + tj + C \qquad v(0) = 0i + 0j + C = i - j$

$\Rightarrow C = i - j$

$$\boxed{v(t) = i(t+1) + j(t-1)}$$

$$\int v(t)dt = \int (ti + i + tj - j)\, dt$$

$$P(t) = \frac{t^2}{2}i + ti + \frac{t^2}{2}j - tj + C$$

$$P(0) = 0 + 0 + 0 - 0 + C = K \Rightarrow C = K$$

$$\boxed{P(t) = i\left(\frac{t^2}{2} + t\right) + j\left(\frac{t^2}{2} - t\right) + K}$$

Similar Problem: Find the position vector of a particle whose acceleration is $a(t) = ti + tj + tK$ and at time $t=0$, velocity is i and position is $3K$.

Answer: $\int a(t) = \int ti + tj + tK \, dt = \frac{t^2}{2}i + \frac{t^2}{2}j + \frac{t^2}{2}K + C = v(t)$

$v(0) = 0 + 0 + 0 + C = i \Rightarrow C = i$

$$\boxed{v(t) = i\left(\frac{t^2}{2} + 1\right) + j\left(\frac{t^2}{2}\right) + K\left(\frac{t^2}{2}\right)}$$

$$P(t) = \int v(t)dt = \frac{t^3}{6}i + ti + \frac{t^3}{6}j + \frac{t^3}{6}K + C$$

$v(0) = 0 + 0 + 0 + 0 + C = 3K \Rightarrow C = 3K$

$$P(t) = i\left(\frac{t^3}{6} + t\right) + j\left(\frac{t^3}{6}\right) + K\left(\frac{t^3}{6} + 3\right)$$

scc9 Rahul Pateja

13.3 → Suppose that the position of a certain particle is given by

$$r(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle \quad 0 \le t \le \pi$$

(a) Find the velocity of the particle as a function of the time t

$u = e^t \quad v = \cos t$
$u' = e^t \quad v' = -\sin t$

$u = e^t \quad v = \sin t$
$u' = e^t \quad v' = \cos t$

$$r'(t) = \langle -e^t \sin t + e^t \cos t, e^t \cos t + e^t \sin t, e^t \rangle$$

$$v(t) = \langle e^t(\cos(t) - \sin(t)), e^t(\cos(t) + \sin(t)), e^t \rangle$$

(b) Find the length of the arc traversed by the moving particle for $0 \le t \le \pi$

$$|r'(t)| = \sqrt{(-e^t \sin t + e^t \cos t)^2 + (e^t \cos t + e^t \sin t)^2 + (e^t)^2}$$

$$\int_0^{\pi} |r'(t)| \, dt = \sqrt{3}\,(e^{\pi} - 1)$$

Similar Problem: Suppose that the position of a certain particle is given by

$$r(t) = \langle 2\sin t, 6t, 2t^2 \rangle \quad 0 \le t \le 2\pi$$

(a) Find the velocity of the particle as a function of time t

$$r'(t) = \langle 2\cos t, 6, 4t \rangle = v(t)$$

(b) Find the length of the arc traversed by the moving particle for $0 \le t \le 2\pi$

$$|r'(t)| = \sqrt{(2\cos t)^2 + 6^2 + 4t^2} = \sqrt{4\cos^2 t + 36 + 4t^2}$$

$$= 2\cos t + 6 + 2t$$

$$\int_0^{2\pi} 2\cos t + 6 + 2t \, dt = 2\sin t + 6t + t^2 \Big]_0^{2\pi}$$

$$= 12\pi + 4\pi^2$$

$$= \boxed{4\pi(3 + \pi)}$$

13.4 → Find the curvature of the curve

$r(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$

at the point $(1, 1, \frac{2}{3})$ $\quad t = 1$

Answer: $r'(t) = \langle 1, 2t, 2t^2 \rangle \quad r''(t) = \langle 0, 2, 4t \rangle$

$r'(1) = \langle 1, 2, 2 \rangle \quad r''(1) = \langle 0, 2, 4 \rangle$

$$r'(1) \times r''(1) = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 0 & 2 & 4 \end{vmatrix} = i(4) - j(4) + k(2)$$

$$= 4i - 4j + 2k$$

$$|4i - 4j + 2k| = \sqrt{16 + 16 + 4} = \sqrt{36} = 6$$

$$|r'(1)| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$\frac{6}{|r'(1)|^3} = \frac{6}{3^3} = \frac{6}{27} = \boxed{\frac{2}{9}}$$

Similar Problem: Find the curvature of the curve $r(t) = \langle 2t^2, t^3, t^4 \rangle$ at the point $(2, 1, 1)$

Answer: $t = 1 \quad r'(t) = \langle 4t, 3t^2, 4t^3 \rangle$

$r''(t) = \langle 4, 6t, 12t^2 \rangle$

$r'(1) = \langle 4, 3, 4 \rangle \quad r'(1) \times r''(1) = \begin{vmatrix} i & j & k \\ 4 & 3 & 4 \\ 4 & 6 & 12 \end{vmatrix}$

$r''(1) = \langle 4, 6, 12 \rangle$

$$i(12) - j(32) + k(12)$$

$$= 12i - 32j + 12k = \sqrt{(12)^2 + (32)^2 + (12)^2} = \sqrt{1312}$$

$$|r'(1)| = \sqrt{16 + 9 + 16} = (\sqrt{41})^3$$

Answer: $\boxed{\frac{\sqrt{1312}}{(\sqrt{41})^3}}$

Scc2 Rahul Paleja

13.5 → None

14.2 → Complete the limit

$$\lim_{(x,y,z)\to(1,1,1)} e^{-xy}\sin\left(\frac{\pi z}{2}\right)$$

Answer: Plug In → $e^{-1\cdot 1}\sin\left(\frac{\pi(1)}{2}\right) = \frac{1}{e}\cdot 1 = \boxed{\frac{1}{e}}$

Similar Problem: Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^3+y^2}$$

Answer: Plug In $\frac{0}{0+0} = \frac{0}{0}$ → limit May or May not exist

Plug In $y = cx$ so $\lim_{x\to 0} \frac{x^2}{x^2+(cx)^2} = \lim_{x\to 0} \frac{x^2}{x^2(1+c)}$

$\lim_{x\to 0} \frac{1}{1+c}$ → This limit depends on slope c, so we get different limits depending on how we approach the point from different lines, thus the limit does not exist.

14.3 → None

14.4 - D Find an equation of the tangent plane to the surface

$$z = e^{2x-3y}$$

at the point $(3,2,1)$. Simplify as much as you can!

Answer: Point works $1 = e^{6-6} = e^0 = 1 = 1 \checkmark$

$\frac{dz}{dx} = 2e^{2x-3y}$ $\quad \frac{dz}{dy} = -3e^{2x-3y}$

Plug in: $2e^0 = 2$ $\quad = -3e^0 \rightarrow -3$

Equation: $z - 1 = 2(x-3) - 3(y-2)$

$$\boxed{z = 2x - 3y + 1}$$

Similar Problem: Find an equation of the tangent plane to the surface

$$z = 2x^2 + 3y^3 - 1$$

at the point $(1,1,4)$. Simplify as much as you can!

Answer: Point works $4 = 2 + 3 - 1$

$\frac{dz}{dx} = 4x$ $\quad \frac{dz}{dy} = 9y^2$

Plug in Point: 4 $\quad \rightarrow$ DO 9

Equation:

$$z - 4 = 4(x-1) + 9(y-1)$$

$$z - 4 = 4x - 4 + 9y - 9$$

$$\boxed{z = 4x + 9y - 9}$$

SCC1 Rahul Paleja

14.5 → Let $f(x,y,z) = -x^2 + y^2 + z^2 - 1$

(a) Compute Df

$Df = \langle -2x, 2y, 2z \rangle$

(b) Find a normal to the level surface $F(x,y,z) = 0$ at the point $(1,1,1)$ and give an equation for the tangent plane to the surface at that point.

$-x^2 + y^2 + z^2 - 1 = 0$ $\quad z^2 = x^2 - y^2 + 1$

$z = x - y + 1$

$dz/dx = 1$ $\quad dz/dy = -1$

Equation:

$z - 1 = 1(x-1) - 1(y-1)$

$\boxed{z = x - y + 1}$

(c) Compute the directional derivative of $F(x,y,z)$ at the point $(1,1,1)$ in the direction $\langle 1,2,2 \rangle$

$Df(1,1,1) = \langle -2,2,2 \rangle \cdot u$

$\langle 1,2,2 \rangle = \sqrt{1+4+4} = 3$ unit vector: $\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$\langle -2,2,2 \rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$= -\frac{2}{3} + \frac{4}{3} + \frac{4}{3} = \frac{6}{3} = \boxed{2}$

<u>Similar Problem:</u> Compute the directional derivative of $f(x,y,z) = 2x + 3y + z^3$ at point $(1,1,5)$ in direction $\langle 1,1,1 \rangle$

$Df = \langle 2, 3, 3z^2 \rangle$ $\quad Df(1,1,5) = \langle 2,3,75 \rangle$

$|\langle 1,1,1 \rangle| = \sqrt{3}$ unit vector: $\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

$\langle 2,3,75 \rangle \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

$= \frac{2}{\sqrt{3}} + \frac{3}{\sqrt{3}} + \frac{75}{\sqrt{3}} = \boxed{\frac{80}{\sqrt{3}}}$

14.6-D Find $\frac{df}{dr}$ and $\frac{df}{ds}$ as functions of r and s, if $f(x,y) = x^3+2xy+y^3$ and the variables are related by $x=r-s$ & $y=r+s$. Do Not need to simplify!

Answer: $\frac{df}{dr} = \frac{df}{dx}\cdot\frac{dx}{dr} + \frac{df}{dy}\cdot\frac{dy}{dr}$

$= (3x^2+2y)\cdot(1) + (2x+3y^2)(1)$

$\boxed{df/dr = 3(r-s)^2 + 2(r+s) + 2(r-s) + 3(r+s)^2}$

$df/ds = df/dx \cdot dx/ds + df/dy \cdot dy/ds$

$= (3x^2+2y)(-1) + (2x+3y^2)(1)$

$\boxed{df/ds = -3(r-s)^2 - 2(r+s) + 2(r-s) + 3(r+s)^2}$

Similar Problem: Find df/dr and df/ds as functions of r and s if $f(x,y) = x^2+y^3$ and $x=r$ and $y=s$. Simplify!

Answer: $\frac{df}{dr} = \frac{df}{dx}\cdot\frac{dx}{dr} + \frac{df}{dy}\cdot\frac{dy}{dr}$

$= 2x(1) + 3y^2(0)$

$= 2x = \boxed{2(r) = df/dr}$

$u=x \quad v=z$
$u'=1 \quad v'=z'$

$5y(xz)$

14.6-D Find dz/dx and dz/dy if $\sin(x+2y+3z) = 5xyz+$

Answer: $\cos(x+2y+3z)(1+3z') = 5y(xz'+z)$

$\cos(x+2y+3z) + 3z'(\cos(x+2y+3z)) = 5yxz' + 5yz$

$\cos(x+2y+3z) - 5yz = z'(5yx - 3\cos(x+2y+3z))$

$\boxed{dz/dx = z' = \frac{\cos(x+2y+3z) - 5yz}{5yx - 3\cos(x+2y+3z)}}$

$dz/dy = \cos(x+2y+3z)(2+3z') = 5x(yz'+z)$

$= 2\cos(x+2y+3z) - 5xz = 5xyz' - 3z'\cos(x+2y+3z)$

$\boxed{dz/dy = z' = \frac{2\cos(x+2y+3z) - 5xz}{5xy - 3\cos(x+2y+3z)}}$

scc9 Rahul Paleja

Similar Problem: Find dz/dx and dz/dy if

$$3\cos(x+3y) = 6xz$$

$u = x \quad v = z$
$u' = 1 \quad v' = z'$

$dz/dx \rightarrow$ $-3\sin(x+3y)(1) = 6(xz' + z)$

$-3\sin(x+3y) - 6z = 6xz'$

$$-\frac{3(\sin(x+3y)+2z)}{6x} = \boxed{\frac{dz}{dx} = -\frac{(\sin(x+3y)+2z)}{2}}$$

$dz/dy \rightarrow$ $-3\sin(x+3y)(3) = 6x(z')$

$$z' = \frac{dz}{dy} = -\frac{9\sin(x+3y)}{6x} = \boxed{-\frac{3\sin(x+3y)}{2x}}$$

14.7 → Find the local maximum + minimum points, the local maximum and minimum values, and saddle points of the function

$$f(x,y) = 4x^2 + y^2 + 2x^2y - 1$$

Answer:

$f_x = 8x + 4xy$
$f_y = 2y + 2x^2$
$f_{xx} = 8 + 4y$
$f_{xy} = 4x$
$f_{yy} = 4$

$f_x = 0 \rightarrow 8x + 4xy = 0$ $\qquad f_y = 0$ $\qquad 2y = -2x^2$

$y = -x^2$

$y = 0, -2,$

$8x + 4x(-x^2) = 0$

$8x - 4x^3 = 0$

$4x(2 - x^2) = 0$

$x = 0, \pm\sqrt{2}$

Critical Points: $(0,0)$ $\quad (\sqrt{2}, -2)$ $\quad (-\sqrt{2}, -2)$

$(0,0)$ $(-\sqrt{2},-2)$ $(\sqrt{2},-2)$

$f_{xx}(0,0) = 8$
$f_{xy}(0,0) = 0$
$F_{yy}(0,0) = 4$
$D = 8(4) - 0^2 = 32$
$(0,0)$ is a local minimum
$F(0,0) = -1 \rightarrow$ minimum value

$f_{xx}(-\sqrt{2},-2) = 0$
$f_{xy}(-\sqrt{2},-2) = -4\sqrt{2}$
$f_{yy}(-\sqrt{2},-2) = 4$
$D = 0 - (-4\sqrt{2})^2$
D is negative
$(-\sqrt{2},2)$ is a saddle point

$f_{xx}(\sqrt{2},2) = 0$
$f_{xy}(\sqrt{2},2) = 4\sqrt{2}$
$f_{yy}(\sqrt{2},2) = 4$
$D = (0 \cdot 4) - (4\sqrt{2})^2 \rightarrow$ Negative
$(\sqrt{2},-2)$ is a saddle point

14.7 → Find the local maximum and minimum point(s), the local max and min values, and saddle point(s) of the function

$$f(x,y) = 6y^2 - 2y^3 + 3x^2 + 6xy$$

Answer: $f_x = 6x + 6y$ $f_y = 12y - 6y^2 + 6x$
$f_{xx} = 6$ $f_{yy} = 12 - 12y$
$f_{xy} = 6$

$f_x = 0$ $6x = -6y$ $x = -y$ $x = 0, -1$

$12y - 6y^2 + 6(-y) = 0$
$6y - 6y^2 = 0$
$6y(1-y) = 0$ $y = 0, 1$

Critical Points: $(0,0)$; $(-1,1)$

$F_{xx}(0,0) = 6$
$F_{yy}(0,0) = 12$
$F_{xy}(0,0) = 6$
$D = 6(12) - 6^2 \rightarrow$ Positive
$(0,0)$ is a local min at 0-value

$f_{xx}(-1,1) = 6$
$f_{xy}(-1,1) = 6$
$f_{yy}(-1,1) = 0$
$D = 0 - 6^2 = -36$
$(-1,1)$ is a saddle point

scc1 Rahul Paleja

Similar Problem: Find the local max, local min, and saddle point(s) of $f(x,y) = x^2 - 3xy + 6y^2 - 3x + 7y + 3$

Answer: $F_x = 2x - 3y - 3$

$F_y = -3x + 12y + 7$

$F_{xx} = 2 \qquad F_{yy} = 12$

$F_{xy} = -3$

$f_x = 0$ & $f_y = 0 \quad 2x - 3y - 3 = 0 \quad x = \frac{3y+3}{2}$ $\boxed{x = 1}$

$-3\left(\frac{3y+3}{2}\right) + 12y + 7 = 0 \rightarrow \frac{-9y-9}{2} + \frac{24y}{2} + 7 = 0$

$\left(\frac{15y-9}{2} = -7\right)2$

$15y - 9 = -14$

$15y = -5$ $\boxed{y = -1/3}$

One critical Point $(1, -1/3)$

$F_{xx}(1, -1/3) = 2 \quad f_{xy}(1, -1/3) = -3 \quad f_{yy}(1, -1/3) = 12$

$D = 2(12) - (-3)^2 = 24 - 9 = 15 > 0$

& $f_{xx} = 2 > 0$ SC

$(1, -1/3)$ is a local min.

Similar Problem: Find the absolute min value and absolute max value of $f(x,y) = x^2 - 9xy + y^2 + 5x + 5y + 3$ over the region $S = \{(x,y) \mid 0 \le x \le 7, 1 \le y \le 9\}$

$F_x = 2x - 9y + 5$

$2x - 9y + 5 = 0$

$x = \frac{9y-5}{2} \quad \frac{9\left(\frac{5}{11}\right) - 5}{2} = \frac{\frac{45}{11} - \frac{55}{11}}{2}$

$x = \frac{-5}{11}$ ◁

Critical Point $\left(-\frac{5}{11}, \frac{5}{11}\right)$

$f_y = -9x + 2y + 5 = 0$

$\frac{-9(9y-5)}{2} + 2y + 5 = 0$

$\frac{-81y + 45 + 4y}{2} = -5$

$-77y + 45 = 10$

$y = \frac{5}{11}$

$f\left(-\frac{5}{11}, \frac{5}{11}\right) = 5.27$

Max

Left side: $x=0 \quad 1 \le y \le 9$

$f(0,y) = y^2+5y+3 = F(y)$

$F'(y) = 2y+5$ at $y=-\frac{5}{2}$

$F(-\frac{5}{2}) = (-\frac{5}{2})^2 + (\frac{5}{1}\cdot(-\frac{5}{2}) + 3 = \frac{+25}{4} - \frac{50}{4} = \frac{-13}{4}$

$F(1)=9 \quad F(9)=129$

abs. Min of left side $=-13/4$ Abs. Max of left side $=129$

Right side: $x=7 \quad 1 \le y \le 9 \quad f(7,y) = 49-(3y+y^2+35+5y+3$

$F(y) = 87-58y+y^2=0 \quad y=29+\sqrt{}$

$F'(y) = -58+2y=0 \quad y=29 \quad F(29)=-754 \quad F(1)=35 \quad F(9)=-354$

Abs. Min of RS $=-754$

Abs. Max of RS $=30$

Down side: $y=1 \quad 0 \le x \le 7 \quad f(x,1) = x^2-9x+1+5+3 = x^2-9x+9=F(x)$

$F'(x) = 2x-4=0 \quad x=2 \quad F(2)=5 \quad F(0)=9 \quad F(7)=30$

Abs. Min of DS $=5$

Abs. Max of DS $=30$

Up side: $y=9 \quad 0 \le x \le 7 \quad f(x,9) = x^2-9x+81+5x+45+3$

$F(x) = x^2-76x+129$

$F'(x) = 2x-76 \rightarrow x=38$

$F(38)=-1315 \quad F(0)=129 \quad F(7)=-354$

Abs. Min of U.S. $=-1315$

Abs. Max of U.S. $=129$

Final Answer: Abs. Max $=129$

Abs. Min $=-1315$

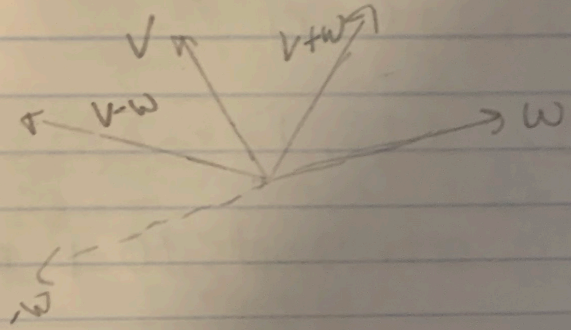
scc1 Rahul Paleja

Exercises From Textbook:

Section 12.1:

(16) $4(\langle 1,1\rangle + \langle 3,2\rangle) \rightarrow 4\langle 4,3\rangle$
$= \langle 16,12\rangle$

(20) Sketch $v+w$ and $v-w$



Section 12.2:

(26) Determine whether or not the 2 vectors are parallel
$u = \langle 4,2,-6\rangle$, $v = \langle 2,-1,3\rangle$
The vectors are not multiples of each other thus are not parallel

(17) Determine whether $\overrightarrow{AB} = \overrightarrow{PQ}$

$\overrightarrow{AB} = \langle 2,2,5\rangle$
$\overrightarrow{PQ} = \langle 2,2,6\rangle$ > No, they are not equivalent

12.3

(22) Find cos of angle between the vectors
$3i+k$, $i+j+k$

vectors: $\langle 3,0,1\rangle = \sqrt{10}$
$\langle 1,1,1\rangle = \sqrt{3}$

$v \cdot w = 3+1 = 4$

$\cos\theta = \frac{v \cdot w}{|v||w|}$

$\boxed{\cos\theta = \frac{4}{\sqrt{30}}}$

(32) Find a vector that is orthogonal to $v=\langle 1,2,1\rangle$ but not $w=\langle 1,0,-1\rangle$

Orthogonal → Dot Product = 0

$\langle -2,1,0\rangle \cdot \langle 1,2,1\rangle$
$= -2+2+0=0$

$\langle -2,1,0\rangle \cdot \langle 1,0,-1\rangle$
$= -2$

$\langle -2,1,0\rangle$ is orthogonal to v but not w

12.4:

(18) $v \times (u+v)$ Distribute:

$(v \times u) + (v \times v)$

$-(u \times v)$ $\quad 0 \rightarrow$ Cross Product of same vector = 0

Answer: $\boxed{\langle -1,-1,0\rangle}$

(16) Calculate Cross Product:

$(2i-3j+4k) \times (i+j-7k)$

$$\begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 1 & 1 & -7 \end{vmatrix}$$

$i(21-4) - j(-14-4) + k(5)$
$= 17i + 18j + 5k$
$= \boxed{\langle 17,18,5\rangle}$

scc1 Rahul Paleja

12.5

(14) $x - z = 0$ → Find a vector normal to the plane with that equation

Normal vector will be coefficients of x, y, z components:

$\boxed{\langle 1, 0, -1 \rangle}$

(20) $P = (2,0,0)$, $Q = (0,4,0)$, $R = (0,0,2)$

$\overrightarrow{PQ} = \langle -2, 4, 0 \rangle$

$\overrightarrow{PR} = \langle -2, 0, 2 \rangle$

$$\begin{vmatrix} i & j & k \\ -2 & 4 & 0 \\ -2 & 0 & 2 \end{vmatrix} \quad i(8) - j(-4) + k(8)$$

$= 8i + 4j + 8k$

$= \langle 8, 4, 8 \rangle$

$8(x-2) + 4(y) + 8(z) = 0$

$8x - 16 + 4y + 8z = 0$

$\frac{8x + 4y + 8z}{4} = \frac{16}{4}$

$\boxed{2x + y + 2z = 4}$

13.1

(6) Find a direction vector for the line with parameterization

$r(t) = (4 - t)i + (2 + 5t)j + \frac{1}{2}tk$

$r(t) = \langle 4, 2, 0 \rangle + t\langle -1, 5, \frac{1}{2} \rangle$

Direction vector: $\langle -1, 5, \frac{1}{2} \rangle$

(18) Determine radius, center, & Plane containing the circle

$r(t) = 7i + (12\cos t)j + (12\sin t)k$

$x(t) = 7 \quad y(t) = 12\cos t \quad z(t) = 12\sin t$

$y^2 + z^2 = (12\cos t)^2 + (12\sin t)^2$

$= 144(\cos^2 t + \sin^2 t) = 144$

so $y^2 + z^2 = 144$ Center: $(7,0,0)$ Radius $= 12$

in yz plane

Scc2 Rahul Paleja

13.2:

(8) Find $r'(t)$

$r(t) = \langle 7-t, 4\sqrt{t}, 8 \rangle$

$r'(t) = \langle -1, \frac{2}{\sqrt{t}}, 0 \rangle$

(32) Find a parameterization of tangent line at the point indicated.

$r(t) = \langle 4t, 5t, 9t \rangle, \; t = -4$

$r(-4) = \langle -16, -20, -36 \rangle$

$r'(t) = \langle 4, 5, 9 \rangle$

$$L(t) = \langle -16, -20, -36 \rangle + t\langle 4, 5, 9 \rangle$$

13.3:

(10) Compute arc length $s(t) = \int_a^t \|r'(u)\| \, du$

For $r(t) = \langle 4t^{1/2}, \ln t, 2t \rangle \; a = 1$

$r'(t) = \langle 2t^{-1/2}, \frac{1}{t}, 2 \rangle$

$\|r'(t)\| = \sqrt{(2t^{-1/2})^2 + (t^{-1})^2 + (2)^2}$

$= \sqrt{(t^{-1}+2)^2} = t^{-1} + 2$

Plug In:

$$\int_a^t (u^{-1}+2)\, du = \ln(u) + 2u \Big]_1^t$$

$$s(t) = \ln(t) + 2t - 2$$

(14) Find speed at given value of t

$r(t) = \langle e^{t-3}, 2, 3t^{-1} \rangle, \; t = 3$

$r'(t) = \langle e^{t-3}, 0, -3t^{-2} \rangle \quad \|r'(t)\| = \sqrt{(e^{t-3})^2 + (-3t^{-2})^2}$

$= \sqrt{e^{2t-6} + 9t^{-4}}$

$v(3) = \sqrt{e^{6-6} + 9(3)^{-4}} = \sqrt{1 + \frac{1}{9}} = \sqrt{\frac{10}{9}}$

$$= \frac{\sqrt{10}}{3}$$

... y (0,0) so limit DNE

13.4:

(12) Find curvature at given point

$r(t) = \langle 3-t, e^{t-4}, 8t-t^2 \rangle \quad t=4$

$r'(t) = \langle -1, e^{t-4}, 8-2t \rangle$

$r'(4) = \langle -1, 1, 0 \rangle$

$r''(t) = \langle 0, e^{t-4}, -2 \rangle$

$r''(4) = \langle 0, 1, -2 \rangle$

$$r'(1) \times r''(1) = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$i(-2) - j(2) + k(-1)$

$= -2i - 2j - k = \langle -2, -2, -1 \rangle$

$|r'(t) \times r''(t)| = \sqrt{(-2)^2 + (-2)^2 + (-1)^2}$

$= 3$

$|r'(t)| = \sqrt{(-1)^2 + (1)^2} = (\sqrt{2})^3$

Answer $\rightarrow K(4) = \frac{3}{(\sqrt{2})^3} = \boxed{\frac{3}{2^{3/2}}}$

(13) Find curvature of plane curve: $y = e^t$ at $t=3$

Formula: $K(t) = \frac{|f''(t)|}{(1+f'(t)^2)^{3/2}}$

$f'(t) = e^t \qquad f''(t) = e^t$

$K(t) = \frac{|e^t|}{(1+e^{2t})^{3/2}}$

$K(3) = \frac{|e^3|}{(1+e^6)^{3/2}} = \boxed{\frac{e^3}{(1+e^6)^{3/2}}}$

which varies depending on the line approaching (0,0) so limit DNE

Scc1 Rahul Paleja

13.5:

(13) Find $v(t)$ given $a(t)$ and initial velocity

$a(t) = k \quad v(0) = i$

$\int a(t) = tK + C = v(t)$

$v(0) = 0 + C = i \qquad C = i$

$\boxed{v(t) = i + tK}$

(16) Find $r(t)$ and $v(t)$

$a(t) = \langle e^t, 2t, t+1 \rangle \quad v(0) = \langle 1,0,1 \rangle \quad r(0) = \langle 2,1,1 \rangle$

$\int a(t) = e^t i + t^2 j + (\frac{t^2}{2} + t)k + C = v(t)$

$v(0) = 1 + C = 1 + K \qquad C = K$

$\int v(t) = \boxed{e^t i + t^2 j + (\frac{t^2}{2} + t + 1)k}$

$r(t) = e^t i + \frac{t^3}{3} j + (\frac{t^3}{6} + \frac{t^2}{2} + t)k + C$

$r(0) = i + C = 2i + j + k$

$C = i + j + k$

$\boxed{r(t) = (e^t + 1)i + (\frac{t^3}{3} + 1)j + (\frac{t^3}{6} + \frac{t^2}{2} + t + 1)k}$

14.2:

(13) Does $\lim_{(x,y) \to (0,0)} \frac{y^2}{x^2 + y^2}$ exist?

Plug in $\to \frac{0}{0}$ let $y = cx$

$\lim_{x \to 0} \frac{(cx)^2}{x^2 + (cx)^2} = \frac{c^2 x^2}{x^2(1 + c^2)} = \frac{c^2}{1 + c^2}$

limit still depends on c which varies depending on the line approaching $(0,0)$ so limit DNE

(28) $\lim_{(z,w)\to(-1,2)} (z^2 w - 9z)$

Plug in: $(-1)^2 \cdot 2 - 9(-1)$

$= 2 + 9 = \boxed{11}$

Section 14.3:

(14) $F_x = 4x^3y^3 \quad F_y = 3y^2x^4$

Original Function $z = x^4y^3$

(37) $w = xy^2z^3$

$F_x = y^2z^3 \quad F_y = 2xyz^3 \quad F_z = 3xy^2z^2$

14.4:

(8) $g(x,y) = e^{x/y}$, $(2,1)$ Find equation of tangent plane at given point

$g_x = \frac{1}{y}e^{x/y} \quad g_x(2,1) = e^2$

$g_y = -\frac{x}{y^2}e^{x/y} \quad g_y(2,1) = -2e^2$

$g(2,1) = e^2$

$z = e^2(x-2) - 2e^2(y-1) + e^2$

$\boxed{z = e^2x - 2e^2y + e^2}$

scc1 Rahul Paleja

(24) 14.4:

use linear approximation to estimate value

$\frac{4.1}{7.9}$ $f(x,y) = \frac{x}{y}$

Use close value:

(4,8) $f_x = \frac{1}{y}$ $f_y = \frac{-x}{y^2}$

$f(4,8) = \frac{1}{2}$ $f_x(4,8) = \frac{1}{8}$ $f_y(4,8) = \frac{-1}{16}$

$f(x,y) = \frac{1}{8} + \frac{1}{2}(x-4) - \frac{1}{16}(y-8)$

$f(4.1,7.9) = \frac{1}{8} + \frac{1}{2}(.1) - \frac{1}{16}(-.1) = .519$

$f(4.1,7.9) = \frac{4.1}{7.9} = .51899$ very close linear approximation works

(14) 14.5:

Calculate $\frac{d}{dt} f(r(t))$ $f(x,y) = \cos(y-x)$, $r(t) = \langle e^t, e^{2t} \rangle$, $t = \ln 3$

$F_x = \langle \sin(y-x), -\sin(y-x) \rangle$ $r(\ln(3)) = \langle e^{\ln(3)}, e^{2\ln(3)} \rangle = \langle 3, 9 \rangle$

$Df_{(3,9)} = \langle \sin 6, -\sin 6 \rangle$

$r'(\ln(3)) = \langle e^{\ln(3)}, 2e^{2\ln(3)} \rangle = \langle 3, 18 \rangle$

$\langle \sin 6, -\sin 6 \rangle \cdot \langle 3, 18 \rangle = 3\sin 6 - 18\sin 6 = \boxed{-15\sin 6}$

(10) $f(x,y) = xe^y$ $\quad r(t) = \langle t^2, t^3 - 4t \rangle, \; t = 0$

$r(0) = \langle 0, 0 \rangle$

$Df = \langle e^y, xe^y \rangle$

$Df_{(0,0)} = \langle e^0, 0 \rangle = \langle 1, 0 \rangle$

$r'(t) = \langle 2t, 2t - 4 \rangle$

$r'(0) = \langle 0, -4 \rangle$

$\langle 1, 0 \rangle \cdot \langle 0, -4 \rangle = \boxed{0}$

14.6:

(34) Find $\frac{df}{dr}, \frac{df}{dt}$ if $f(x,y,z) = xy + z^3$, $x = r + s - 2t$, $y = 3rt$, $z = s^2$

$f_x = y \quad f_y = x \quad f_z = 2z$

$\frac{dx}{dr} = 1 \quad \frac{dx}{dt} = -2$

$\frac{dy}{dr} = 3t \quad \frac{dy}{dt} = 3r$

$\frac{dz}{dr} = 0 \quad \frac{dz}{dt} = 0$

$\frac{df}{dr} = y + 3xt$

$= 3rt + 3t(r + s - 2t)$

$= 3rt + 3rt + 3ts - 6t^2$

$\frac{df}{dr} = \boxed{6rt + 3st - 6t^2}$

$\frac{df}{dt} = y(-2) + x(3r)$

$= -2y + 3r(x)$

$= -2(3rt) + 3r(r + s - 2t)$

$= -6rt + 3r^2 + 3rs - 6rt$

$\frac{df}{dt} = \boxed{3r^2 + 3rs - 12rt}$

(8) $\frac{df}{du}$ if $f(x,y) = x^2 + y^2$; $x = e^{u+v}$, $y = u + v$

$\frac{df}{dx} = 2x \quad \frac{df}{dy} = 2y \quad \frac{dx}{du} = e^{u+v} \quad \frac{dy}{du} = 1$

$\frac{df}{du} = 2x \cdot e^{u+v} + 2y$

$= 2(e^{u+v}) \cdot e^{u+v} + 2(u + v)$

$= \boxed{2e^{2u+2v} + 2u + 2v}$

Sec 2 Rahul Paleja

14.7:

(8) Find critical points, local min, local max, or saddle points

$f(x) = x^3 - xy + y^3$

$f_x = 3x^2 - y$ $\quad$ $f_y = -x + 3y^2 = 0$

$f_x = 0$ $\quad$ $y = 3x^2$

$-x + 3(9x^4) = 27x^4 - x = 0$

$x(27x^3 - 1) = 0$

$x = 0, \frac{1}{3}$

$y = 0, \frac{1}{3}$

Critical Points: $(0,0), (\frac{1}{3}, \frac{1}{3})$

$f_{xx} = 6x$

$f_{xy} = -1$

$f_{yy} = 6y$

At Point $(0,0)$

$D = 0 \cdot 0 - (-1)^2 = -1$ $\quad \rightarrow (0,0)$ = saddle point

At $(\frac{1}{3}, \frac{1}{3})$

$D = 2 \cdot 2 - 1 = 3$ + $f_{xx} = 2$

so $(\frac{1}{3}, \frac{1}{3})$ is a local min

(16) $f(x) = e^x - xe^y$

$f_x = e^x - e^y$ $\quad$ $f_y = -xe^y = 0$

$f_x = 0$

$e^x = e^y$

$x = y$

$-x e^x = 0$

$x = 0$ $\quad y = 0$

Critical Point $(0,0)$

$f_{xx} = e^x$ $\quad f_{xy} = -e^y$

$f_{yy} = -xe^y$

$D = 1(0) - (-1)^2 = -1$ so $(0,0)$ is a saddle point