11/8/20 Second Chance Club.

Lecture 1 → 12.1 & 12.2.

- Problems from Prev Final: N/A.
- 12.1 Exercises

29) A(1,1), B(3,7), P(4,-1), Q(6,5)
$\langle 2,6 \rangle$ $\langle 2,6 \rangle$
Vectors $\vec{AB}$ and $\vec{PQ}$ are equivalent.

33) A(1,1) B(3,4), P(1,1), Q(7,10)
$\vec{AB}$ $\langle 2,3 \rangle$ $\vec{PQ}$ $\langle 6,9 \rangle$
$\vec{AB} = 3\vec{PQ}$
These vectors are parallel in the same direction.

- 12.2 Exercises

27) $u = \langle -3,1,4 \rangle$, $v = \langle 4,-2,8 \rangle$
↳ These vectors are ~~not~~ parallel since v cannot be rewritten with a scalar coefficient of u.

37) P(1,1,1) and (3,-5,2)
$\vec{PQ}$ (2, -6, 1) =
(2, -6, 1) + $t\langle 3,-5,2 \rangle$ = $\langle 2+3t, -6-5t, 1+2t \rangle$

Lecture 2 → 12.3 & 12.4

- Problems from a previous final: N/A

12.3 Exercises.

15) $\langle 1,2,1 \rangle \cdot \langle 7,-3,-1 \rangle = 7 - 6 - 1 = 0$.
With a dot product of 0, these vectors are orthogonal.

$4+4+1$

10) $\langle 1,1,1\rangle \cdot \langle 2,-1,2\rangle = 2-1+2=3$

$\cos\theta = \frac{3}{\sqrt{3}\cdot 3} = \frac{1}{\sqrt{3}} = 54.73°$

12.4 Exercises:

10) $\langle 2,0,0\rangle \times \langle -1,0,1\rangle$

i	j	k
2	0	0
-1	0	1

$i(0) - j(2) + k(0)$

$= \langle 0,-2,0\rangle$.

17) $v \times u$

~~u×v~~ $u \times v = \langle 1,1,0\rangle$ $u \times w = \langle 0,3,1\rangle$, $v \times w = \langle 2,-1,1\rangle$

$u \times v = -v \times u \quad = \langle -1,-1,0\rangle$

Lecture 3 → 12.5

Previous Final: Plane equation passing through $(1,0,2)$

with line $r(t) = \langle 1,1,1\rangle + t\langle 1,-1,0\rangle$

line: $\langle 1+t, 1-t, 0\rangle$

When $t=0$, $Q = \langle 1,1,0\rangle$

$\vec{PQ} = \langle 0,1,-2\rangle$

i	j	k
0	1	-2
1	-1	0

$i(2) - j(-2) + k(-1)$

$\langle 2,2,-1\rangle$.

$2(x-1) + 2(y) - z(z-2) = 0$

$2x - 2 + 2y + z + 3 = 0$

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12.5 Problems.

19) $P(1,0,0)$, $Q(0,1,1)$, $R(2,0,1)$

$PQ = \langle -1,1,1 \rangle$

$PR = \langle 1,0,1 \rangle$.

$PQ \times PR$ $\quad \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ $\quad \langle 1,2,-1 \rangle$.

$1, 2, 1 \rangle$

$\langle 1,2,-1 \rangle \cdot \langle 0,1,1 \rangle = 0 + 2 - 1 = 1$

$x + 2y - z = 1$.

25) Passes through $(-2,-3,5)$ with normal $\langle 1,0,1 \rangle$.

$= -2 + 5 = d = 3$.

$x + z = 3$

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Lecture 4: 13.1 & 13.2

13.1 Previous Exam Problem: N/A

13.1 Exercises

17) $r(t) = (9\cos t)i + (9\sin t)j$

$r = 9$, centered at $(0,0)$.

X-Y Plane.

28) a) $x^2 + y^2 = z^2$

Let $x = z\cos\theta$, $y = z\sin\theta$

$x^2 + y^2 = z^2\cos^2\theta + z^2\sin^2\theta = z^2$

13.2: Previous Final:

$a(t) = \langle 1, 1, 0 \rangle$

$t = 0$, $v(t) = \langle 1, -1, 0 \rangle$

$t = 0$, $x(t) = \langle 0, 0, 1 \rangle$.

$\int a(t)\, dt \rightarrow \langle t, t, 0 \rangle + C$

$v(0) = \langle 0, 0, 0 \rangle + C = \langle 1, 1, 0 \rangle$. $C = \langle 1, 1, 0 \rangle$

$v(t) = \langle t + 1, t + 1, 0 \rangle$

$\int v(t)\,dt = \langle \frac{t^2}{2} + t, \frac{t^2}{2} + t, 0 \rangle$

$x(0) = \langle \frac{0^2}{2} + 0, \frac{0^2}{2} + 0, 0 \rangle + C = \langle 0, 0, 1 \rangle$

$C = \langle 0, 0, 1 \rangle$

$x(t) = \langle \frac{t^2}{2} + t, \frac{t^2}{2} + t, 1 \rangle$

13.2 Exercises

13) $r(t) = \langle t, t^2, t^3 \rangle$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

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e^{t}

$e^{2t}\cos^2 t - e^{2t}\sin^2 t + e^{2t}\cos^2 t + e^{4}\sin^4 t$

e^{2t}

25) $r(t) = \langle e^t, e^{2t}, 4 \rangle$, $g(t) = 4t + 9$

$\frac{d}{dt} r(g(t)) = \frac{d}{dt} \langle e^{4t+9}, e^{8t+18}, 4 \rangle$

$= \langle 4e^{4t+9}, 8e^{8t+18}, 4 \rangle$

*Lecture 5: 13.3, 13.4, 13.5

$\boxed{\text{13.3 Previous Exam}}$

$r(t) = \langle e^t\cos t, e^t\sin t, e^t \rangle$, $0 \le t \le \pi$

a) $v(t) = \langle e^t\cos t - e^t\sin t, e^t\cos t + e^t\sin t, e^t \rangle$

b) $\int_0^{\pi} \|v(t)\| dt = \sqrt{3}(e^{\pi} - 1)$

13.3 Exercises

19) $r'(t)$: If $\int(r'(t))\,dt = 0$, the bee will be located at the origin/0 vector, $(0,0)$. $\int_0^T \|r'(t)\|$ represents the length of the bee's journey from $0 - T$.

15) $r(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle$, $t = \frac{\pi}{2}$

$r'(t) = \langle \cos 3t \cdot 3, -4\sin 4t, -5\sin 5t \rangle$

$\|r'(t)\| = \sqrt{9\cos^2 3t + 16\sin^2 4t + 25\sin^2 5t}$

$\|r'(\frac{\pi}{2})\| = 5$

$\boxed{\text{13.4 Previous Exam}}$

Curvature of $(r(t)) = \langle t, t^2, \frac{2}{3}t^3 \rangle$ at $(1, 1, \frac{2}{3})$ $t=1$

$r'(t) = \langle 1, 2t, 2t^2 \rangle$ ~~$r'(t) \times r''(t)$ at $(1, 1, \frac{2}{3})$~~

$r''(t) = \langle 1, 2, 2t \rangle$

$$\begin{vmatrix} i & j & k \\ t & 2t & 2t^2 \\ 1 & 2 & 4t \end{vmatrix} \quad \cancel{i\,((4t^2 - 2t^2)) + j\,((2t^2 - t^2)) + k\,(2t - 2t)}$$

$\cancel{t\,2t^2\,(\,) + t^2(\,)}$

$\cancel{\text{Magnitude} = \sqrt{4t^4 + t^4} = \sqrt{5t^4} = t^2\sqrt{5}}$

$= \langle 4t^2 - 4t,\ 2 \rangle$ when $t = 1$,

Mag $= 6$.

$\|r'(t)\| = \sqrt{1 + 4t^2 + 4t^4 + t^4 - 2t^2 + 1}$

$\|r'(1)\| = 3$

$$\frac{6}{3^3} = \frac{2}{9}$$

13.4 Exercises.

15) $y = e^t \quad t = 3$

$y' = e^t \quad y'' = e^t$

$$k(t) = \frac{\|f''(t)\|}{(1 + f'(t)^2)^{\frac{3}{2}}} = \frac{e^3}{(1 + e^{2t})^{\frac{3}{2}}} = \frac{e^3}{(1 + e^6)^{\frac{3}{2}}}$$

19) $r(t) = \langle 2\sin t, \cos 3t, t \rangle$

$r'(t) = \langle 2\cos t, -3\sin 3t, 1 \rangle$

$r''(t) = \langle -2\sin t, -9\cos 3t, 0 \rangle$

$$r'(t) \times r''(t) \quad \begin{pmatrix} i & j & k \\ 2\cos t & -3\sin 3t & 1 \\ -2\sin t & -9\cos 3t & 0 \end{pmatrix}$$

Cross: < ~~8~~ $9\cos 3t$, $-2\sin t$, $(-18\cos t\cos 3t - 6\sin t\, 3t)$ >

$$K(t) = \frac{\sqrt{81\cos^2 3t + 4\sin^2 t + (-18\cos 3t\cos 3t - 6\sin t\, 3t)^2}}{\left(\sqrt{4\cos^2 t + 9\sin^2 3t + 1}\right)^3}$$

$K\left(\frac{\pi}{3}\right) = 4.541$

$K\left(\frac{\pi}{2}\right) = \frac{1}{2}$

$\boxed{\text{13.5 : Previous Exam: NA.}}$

13.5 Exercises.

15) $a(t) = \langle t, 4 \rangle$ $v(0) = \langle 3, -2 \rangle$, $r(0) = (0,0)$

$\int a(t)\,dt = \langle \frac{t^2}{2}, 4t \rangle + C = \langle 3, -2 \rangle$ at $t = 0$.

$C = \langle 3, -2 \rangle$

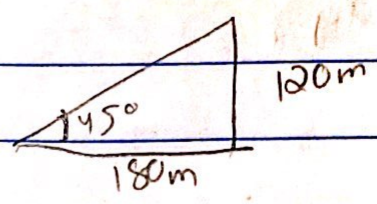
$v(t) = \langle \frac{t^2}{2} + 3, 4t - 2 \rangle$

$\int v(t)\,dt = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$

$r(0) = \langle 0, 0 \rangle + C = \langle 0, 0 \rangle$, $C = 0$

$r(t) = \langle \frac{t^3}{6} + 3t, 2t^2 - 2t \rangle$.

19).



$a(t) = -9.8\, j$

$v(t) = -9.8t + C$

$v(0) = C_0$

$v(0) = v_0 \cos 45°\, i + v_0 \sin 45°\, j$

$v(t) = v_0 \cos 45°\, i + v_0 \sin 45° - 9.8t$.

$\int v(t)\,dt = \langle v\cos 45 t, v\sin 45° t - 4.9t^2 \rangle + C$.

~~$a \cdot \frac{\sqrt{2}}{2} = 180$~~ $(a \cdot \cos 45°) t = 180$

$v \cdot \frac{\sqrt{2}}{2} t = 180$

$\boxed{t = \frac{180\sqrt{2}}{v_0}}$ → Substitute → $v = 72.746$ m/s.

$\boxed{\text{Lecture 6}}$ → 14.1 & 14.2

$\boxed{\text{14.1 → No Previous Exam}}$

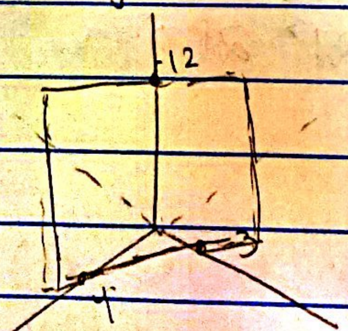
14.1 Exercises

15) $P(r, s, t) = \sqrt{16 - r^2 s^2 t^2}$

$P(r,s,t) \leq 4$ for range to be real.

Range is $0 \leq P \leq 4$.

21) $f(x, y) = 12 - 3x - 4y$



$f(x, y) = 12 - 3x - 4y$.

$\boxed{\text{14.2 Previous Exam}}$

$$\lim_{(x,y,z) \to (1,1,1)} e^{-xy} \sin(\pi z/2)$$

↳ $\frac{1}{e} \cdot \sin(\frac{\pi}{2}) = \frac{1}{e}$

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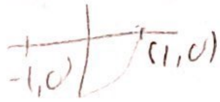

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Lecture 7 14.3 and 14.4

$\boxed{\text{14.3: Previous Exam: NA.}}$

14.3 Exercises:

35) $z = \sinh(x^2 y)$

$dz/dx = 2x\cosh(x^2 y)$

$dz/dy - \cos(x^2 y)$

42) $f(x,y) = \sin(x^2 - y)$ at $f_y\ (0, \pi)$

$f_y - \cos(x^2 - y) \rightarrow \cos(0 - \pi) = -1$.

$\boxed{\text{14.4: Previous Exam}}$

$z = e^{2x-3y}$ at $(3,2,1) \rightarrow$ Find equation of tangent plane.

$dz/dy = -3e^{2x-3y} = -3e$

$dz/dx = 2e^{2x-3y} = 2$

$2(x-3) - 3(y-2) = z-1$

$= 2x - 3y - z = -1$

14.4 Exercises

10) $f(x,y) = \ln(4x^2 - y^2)$, $(1,1)$

$f_x = \frac{8x}{4x^2 - y^2} = \frac{8}{4-1} = 8/3$

$f_y = \frac{2y}{4x^2 - y^2} = \frac{2}{4-1} = \frac{2}{3}$

$z = \ln(3) + \frac{8}{3}(x-1) + \frac{2}{3}(y-1)$

15) $f(x,y) = x^3 y^{-4}$ | $f(2,1) = 8$

$f_x = 3x^2 y^{-4}$ $\quad f_x(2,1) = 12$

$f_y = -4 \cdot x^3 y^{-5}$ $\quad f_y(2,1) = -32$

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$f(2.03, 0.9) - f(2,1)$

$12(0.03) - 32(0.1) = 3.56$

~~ϕ~~ lecture 8 $\rightarrow$ 14.5.

$\boxed{\text{14.5 Previous Final}}$

$f(x,y,z) = -x^2 + y^2 + z^2 - 1$

a) $\nabla f = \langle -2x, 2y, 2z \rangle$

b) $\nabla f_{(1,1,1)} = \langle -2, 2, 2 \rangle$

$2(z-1) = -2(x-1) + 2(y-1)$

$z = x - y + 1$

c) $\| 1,2,2 \| = 3$

$\langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$\nabla f_{(1,1,1)} = \langle -2, 2, 2 \rangle$

$\langle -2, 2, 2 \rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$-\frac{2}{3} + \frac{4}{3} + \frac{4}{3}$

$= 2$

14.5 Exercises.

12) $f(x,y) = x^2 - 3xy \quad r(t) = \langle \cos t, \sin t \rangle \quad t = \frac{\pi}{2}$ $= \langle 0, 1 \rangle$

$f_x = 2x - 3y$

$f_y = -3x$

$\nabla f_{(0,1)} = \langle -3, 0 \rangle$

$r'(t) = \langle -\sin t, \cos t \rangle$

$r'(\pi/2) = \langle -1, 0 \rangle$.

$\nabla f_{r(\pi/2)} \cdot r'(\pi/2) = \langle -3, 0 \rangle \cdot \langle -1, 0 \rangle = 3$

21) $f(x,y) = x^2 + y^3$ $\quad v = \langle 4,3 \rangle$ $\quad P = (1,2)$

$\nabla f = \langle 2x, 3y^2 \rangle$.

$\nabla f_{(1,2)} = \langle 2, 12 \rangle$.

$= \frac{1}{\langle 4,3 \rangle} \langle 2, 12 \rangle \cdot \langle 4, 3 \rangle$

$= \frac{44}{5}$

Lecture 9 → 14.6

Previous Final 14.6

1) $f(x,y) = x^3 + 2xy + y^3$ $\quad \nabla F = \langle 3x^2 + 2y, 2x + 3y^2 \rangle$

$x = r - s$, $y = r + s$, find $\frac{df}{dr}$ and $\frac{df}{ds}$

$\frac{df}{dr} = 3x^2 + 2y + 2x + 3y^2$

$3(r-s)^2 + 2(r+s) + 3(r+s)^2 + 2(r-s)$

$df/ds = -3(r-s)^2 - 2(r+s) + 3(r+s)^2 + 2(r-s)$.

2) dz/dx and dz/dy of $\sin(x + 2y + 3z) = 5xyz + 1$

$dz/dx = \frac{3dz}{dx} \cos(x + 2y + 3z) = 5xy \cdot dz/dx + 5yz$

$\frac{dz}{dx} = \frac{5 - \cos(x + 2y + 3z)}{3\cos(x + 2y + 3z} - 5xy$.

$dz/dy = 2 \frac{dz}{dy} \cos(x + 2y + 3z) = 5xz + 5xy \cdot \frac{dz}{dy}$

$= \frac{5xz - 2\cos(x + 2y + 3z)}{3\cos(x + 2y + 3z)} - 5xy$.

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14.6 Exercises

11) $u,v = (-1,-1)$ $\qquad x = 1-1=0 \quad y = -1+1=0 \quad z=1$

$f(x,y,z) = x^3 + yz^2$ $\qquad x = u^2+v, \; y = u+v^2, \; z = uv$

$\nabla f = \langle 3x^2 + z^2, 2zy \rangle$

$\frac{df}{du} = 3x^2(2u) + z^2(1) + 2zy(v)$

$= 3(0^2) + 1 + 0$

$df/dv = 1$

$df/dz = 3x^2(1) + 2v(z^2) + 2zy(u)$

$= 2(-1)(1) = -2$

17) Total distance: 10.

Hiker: $\frac{8}{10}$, Baseman: $\frac{6}{10}$

$\uparrow$ 20 ft/s $\qquad$ $\uparrow$ 18 ft/s

$\frac{8}{10}(20) + \frac{6}{10}(18)$

$= 134/5$

ft/s.

Lecture 10 $\to$ 14.7

$\boxed{\text{Previous Exam Final}}$

1) $f(x,y) = 4x^2 + y^2 + 2x^2y - 1$.

$\to$ Find min, max (local) and saddle.

$f_x = 8x + 4xy$

$f_y = 2y + 2x^2$ $\qquad 8x + 4xy = 0$

$f_{xx} = 8 + 4y$

$f_{yy} = 2$ $\qquad y + x^2 = 0$.

$f_{xy} = 4$ $\qquad 8x + 4x(-x^2) = 0$

$\cancel{4xy^2}$ 5 $4x^3 - 8x = 0$ $x = 0$

$x^3 - 4 = 0.$

$x = \pm\sqrt{2}$

$y = 0$ or $2y + 2 = 0,$ $y = -1$

Discriminant Calculation

$f_{xx}(0,0) = 8$ $f_{xy}(0,0) = 4$ $f_{yy}(0,0) = 2$

$f_{xx}(\sqrt{2}, 1) = 12$ $f_{xy}(\sqrt{2}, 1) = 4$ $(\sqrt{2}, 1) = 2$

$f_{xx}(\sqrt{2}, -1) = 4.$ $f_{xy}(\sqrt{2}, -1) = 4$ $(\sqrt{2}, 1) = 2$

$f(0,0) \to$ Local Min Point $\to$ Value $= -1.$

$f(\sqrt{2}, -2)$ and $(-\sqrt{2}, -2)$ are saddle points.

2) $f(x,y) = 6y^2 - 2y^3 + 3x^2 + 6xy$

$f_x = 6x + 6y$

$f_y = 12y - 6y^2 + 6x$

$f_{xx} = 6$

$f_{xy} = 6$

$f_{yy} = 12 - 12y$

$6x + 6y = 0$

$(0,0)$ and $(-1,1)$.

$6(0) + 6 = 6 \to (0,0)$ is a local min.

$6(-1) + 6 = 0 \to (-1,1)$ is a saddle point.

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14.7 Exercises

13) $f(x,y) = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y$

$f_{xx} = 12x^2$

$f_{xy} = -4$

$f_y = 4y^3 - 4x$

$f_{yy} = 12y^2$

$4x^3 - 4y = 0$

$x^3 - y = 0$

$y = x^3$

$4x^9 - 4x = 0$

(0,0)
(1,1)
(-1,1)

Crit Points.

D

$(0,0)$ $\quad 12(0)^2 \cdot (12(0)^2) + -4 = -4$

$(1,1)$ $\quad 12(1) \cdot 12(1) - 4 > 0$

$(-1,-1)$ $\quad 12(-1) \cdot 12(-1) - 4 > 0$

Local Max at (0,0), Local Min at (1,1) and (-1,-1).

24) $f(x,y) = x^2$

$f_x = 2x$

$f_y = 0$

Because $f_y = 0 = 0$, y can be any real number as long as $x = 0$.

$f_{xx} = 2$

$f_{xy} = 0$

$f_{yy} = 0$

Thus, $D=0$ for all cases. Therefore, the min. value of $f = 0$, but there is no local maxima.