Problem from previous Final

Lecture (13)

$r(0) = \langle 1, 1, 1 \rangle$

$1(x-1) + 1(y-0) + 1(z-2)$

$x - 1 + y + z - 2$

$x + y + z = 3$

My own problem:

Find equation for (1,0,1) that contans

$r(t) = \langle 2,2,2 \rangle + t\langle 0,0,0 \rangle$

$\langle 2,2,2 \rangle$ at $r(0)$

$2(x-1) + 2(y) + 2(z-1)$

$2x-2+2y+2z-2$

$2x+2y+2z=4$

$x+y+z=2$

Lecture 4

$a(t) = i + j$

$\int i + j$

$C + \quad ti + tj = i - j$

$C = i - j$

$ti + tj + i - j$

$v(t) = (t+1)i + (t-1)j$

$\int ti + tj + i - j$

$\frac{t^2}{2}i + \frac{t^2}{2}j + ti - jt + K$

$(\frac{t^2}{2} + t)i + (\frac{t^2}{2} - t)j + K$

$r(t) = (t^2/2 + t)i + (t^2/2 - t)j + K$

My own problem:

$a(t) = t^3 i + t j$

$t = 0 \rightarrow$ position: i velocity: ti

Find velocity and position

$\int a(t)$

$\frac{t^4}{4} i + \frac{t^2}{2} j + C$

$\int v(t)$

$\frac{t^5}{20} i + \frac{t^3}{6} j + ti + C$

$ti = C$

$v(t) = \frac{t^4}{4} i + \frac{t^2}{2} j + ti$

$r(t) = \frac{t^5}{20} i + \frac{t^3}{6} j + \frac{t^2}{2} i$

Lecture 5

1.

a) $r'(t)$

$\langle e^t(\cos t - \sin t), e^t(\sin t + \cos t), e^t \rangle$

b)

$(\cos - \sin t)(\cos t - \sin)$

$\int_0^{\pi} \sqrt{e^{2t}(\cos - \sin t)^2 + e^{2t}(\sin + \cos t)^2 + e^{2t}}$

$\cos^2 - 2\cos t \sin$
$\sin^2 + 2\sin \cos t$
$+ \cos^2$

$\int_0^{\pi} e^t \sqrt{\cos^2 + \sin^2 + \cos^2 + \sin^2 + 1}$

$\int_0^{\pi} \sqrt{3} e^t$

$\sqrt{3}(e^{\pi} - e^0)$

$\sqrt{3}(e^{\pi} - 1)$

My own problem:

Position: $r(t) = \langle \cos t, t^2 \sin t, e^{t+3} \rangle$

$0 \leq t \leq 2\pi$

Find velocity at t:

$r'(t) = \langle -\sin t, 2t \cos t, e^{t+3} \rangle$

Length of arc

$$\int_0^{2\pi} \sqrt{\sin t^2 + 4\cos^2 t + e^{2t+6}}\, dt$$

$$\int_0^{2\pi} \sqrt{1 + e^{2t+6}}\, dt$$

$$e^{t+3} \Big|_0^{2\pi}$$

$$e^{2\pi+3} - e^3$$

2.

$r'(t) = \langle 1, 2t, 2t^2 \rangle$

$r''(t) = \langle 0, 2, 4t \rangle$

$i \quad j \quad k$

$1 \quad 2t \quad 2t^2$

$0 \quad 2 \quad 4t$

$(8t^2 - 4t^2)i - (4t)j + (2)k$

$\sqrt{(4t^2)^2 + (4t)^2 + (4)}$

$\sqrt{1 + 4t^2 + 4t^4}$

$= \dfrac{\sqrt{16 + 16 + 4}}{\sqrt{1 + 4 + \frac{64}{9}}} = \dfrac{\sqrt{36}}{\sqrt{\frac{162}{9}}}$

$\frac{16}{81}$

$\frac{416}{81}$

$\frac{54}{81}$

$= \dfrac{2}{9}$

Find the curvature of the curve.

$r(t) = \langle t, t^2, t^3 \rangle$

at $(1,1,1)$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

$$\begin{array}{ccc} i & j & k \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{array}$$

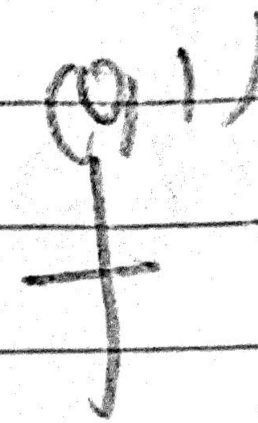
$12t^2 - 6t^2$

$6t^2 i - 6tj + 2k$

$1+4+9$

$$\frac{\sqrt{36+36+4}}{\sqrt{14}} = \frac{\sqrt{76}}{\sqrt{14}}$$

Lecture 6



$e^{-1}\sin(\frac{\pi}{2})$

e^{-1}

$\frac{1}{e}$ ans

My own problem

$$\lim_{(x,y,z)\to(1,1,2)} e^{(x+y)z}$$

$$e^{(1+1)2} = e^{4}$$

Lecture 7

$0 = e^{2x-3y} - z \quad (3,2,1)$

$F_x(3,2,1) = 2e^{2x-3y} = 2e^{6-6} = 2$

$F_y(3,2,1) = -3e^{2x-3y} = -3e^{6-6} = -3$

$F_z(3,2,1) = -1 = -1$

$2(x-3) - 3(y-2) - 1(z-1) = 0$

$2x - 6 - 3y + 6 - z + 1 = 0$

$2x - 3y - z = -1$

My own problem

Find an equation of target plane to $z = 3x^2 + 4y^2$ at $(1, 2, 2)$

$0 = 3x^2 + 4y^2 - z$

$F_x = 6x = 6$

$F_y = 8y = 16$

$F_z = -1$

$6(x-1) + 16(y-2) + 16(z-2)$

$6x - 6 + 16y - 32 + z - 2$

$6x + 16y + z - 40$

Lecture 8

a)

$\langle -2y, 2y, 2z \rangle$

$\langle -4, 2, 2 \rangle$

b) $-2(x-2) + 2(y-1) + 2(z-1)$

$-2x + 4 + 2y - 2 + 2z - 2$

$-2x + 2y + 2z$

$\frac{-2x}{-2} + \frac{2y}{2} + \frac{-2}{-2} = \frac{-2z}{-2}$

$x + y + 1 = z$

c)

$\langle -2x, 2y, 2z \rangle$

$\sqrt{4+4+1} = \sqrt{9} = 3$

$\langle -2, 2, 2 \rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$-\frac{2}{3} + \frac{4}{3} + \frac{4}{3} = \frac{6}{3} = 2$

My own problem

Let $f(x,y,z) = x^3 + \frac{y^2}{2} + z^2 + 6$

d) compute ∇f

$$\langle 3x^2, y, 2z \rangle$$

b) Find a normal to level surface $f(x,y,z) = 0$ at point $(1,1,1)$ and give an equation

$\langle 3, 1, 2 \rangle$ $3(x-1) + 1(y-1) + 2(z-1)$

$3x - 3 + y - 1 + 2z - 2$

$3x + y + 2z = 6$

c) compute directional derivative of $f(x,y,z)$ at $(1,1,1)$ in direction $\langle 3,3,3 \rangle$

$9+9+9$

$\langle 3,1,2 \rangle \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ $\sqrt{27}$

$\sqrt{9}\sqrt{3}$

$\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}$ $3\sqrt{3}$

$\frac{6}{\sqrt{3}}$

Lecture 9

$$\frac{\partial f}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$$

$(3x^2+2y)(+s)+(2x+3y^2)(1)$

$$\frac{\partial f}{\partial r}=3(r-s)^2+2(r+s)+3(r+s)^2+2(r-s)$$

$$\frac{\partial f}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

$\cancel{(3x^2+2y)(-1)+(2x)\,3y^2}$

$$\frac{\partial f}{\partial s}=3(r-s)^2-2(r+s)+3(r+s)^2+2(r-s)$$

2

$\sin(x+2y+3z)-5xyz-1=0$

$$\frac{\partial z}{\partial x}=\frac{-(-5yz+\cos(x+2y+3z))}{3\cos(x+2y+3z)-5xy}$$

$$\frac{\partial z}{\partial y} = \frac{-5xz + 2\cos(x+2y+3z)}{3\cos(x+2y+3z) - 5xy}$$

My own Prolem:

$$f(x,y) = 3x^2 + 4y^2$$

$$x = 2r - s$$

$$y = 3r + s$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} = 12(2r-s) + 24(3r+s)$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} = -6(2r-s) + 8(3r+s)$$

My own problem:

$$\sin(2x+3) = 6x^2 + 5 + y + z^2$$

$$\sin(2x+3) - 6x^2 - 5 + y - z^2$$

$$\frac{\partial z}{\partial y} = \frac{1}{-2z}$$

$$\frac{\partial z}{\partial x} = \frac{-2\cos(2x+3) + 12x}{-2z} = \frac{\cos(-2x+3) + 12x}{z}$$

Lecture 10

$f_x = 8x + 4xy$

$f_y = 2y + 2x^2$

$f_{xx} = 8 + 4y$

$f_{xy} = 4$

$f_{yy} = 2$

$D = 16 - 16 = 0$

$y = -x^2$

$8x + 4(-x^3)$

$8x - 4(x^3)$

$x(8 - 4x^2)$

$x^2 = 2$

$x = \pm\sqrt{2}, 0$

(0, 0) - local minimum

$(\sqrt{2}, -2)$ > saddle point

$(-\sqrt{2}, -2)$

$(8 + 4(-2))(2) - 16 < 0$

$32 - 16$

$D > 0$

$f_{xx} > 0$

2. $f_x = 6x + 6y$

$x = -y$

$y = -x$

$f_{xx} = 6$ $\quad f_{xy}$

$f_y = 12y - 6y^2 + 6x$

$f_{yy} = 12 - 12y$

$-12x - 6x^2 + 6x$

$-6x^2 - 6x$

$x(-6x - 6)$ $\quad (0,0)$

$(-1,1)$

$x = 0$

$x = -1$

$f_{xx}f_{yy}$

$(6)(12) -$

$(6)(12) - (6) < 0$

(0,0) — local min
— value 0

(-1,1) — saddle point

My Own problem

$f(x,y) = 4x^2 + 8y^3$

$f_x = 8x$

$f_{xx} = 8$

$f_{xy} = 0$

$f_y = 24y^2$

$f_{yy} = 12y$

(0,0) - none

$f_{xx}f_{yy} - f_{(xy)}$

$0 - 0 = 0$

$D = 0$

My own problem:

$f(x,y) = 6x^2 + 6xy^2 + 2$

(-3,6) - local min

$f_x = 12x + 6y$

$f_{xx} = 12$

$f_{xy} = 6$

$f_y = 6x$

$f_{yy} = 0$

$0 - 36 < 0$

$D < 0$

$12x = -6y$

$\frac{12x}{-6} = \frac{-6y}{-6}$

$y = -2x$

Second Chance Club

Lecture 1 Homework Problem

12.1

1. Question 42

$\left\langle \frac{24}{25}, \frac{7}{25} \right\rangle$ $\sqrt{24^2+7^2}$ $=\sqrt{625}$

$=25$

2. Question 50

$3v + \langle 5,20 \rangle = \langle 11,17 \rangle$

$3v = \left\langle \frac{6}{3}, \frac{-3}{3} \right\rangle$

$v = \langle 2, -1 \rangle$

$v = \langle 2, -1 \rangle$

12.2

1. Question 34

$r(t) = \langle 4,0,8 \rangle + t\langle 1,0,1 \rangle$

$r(t) = \langle 4+t, 0, 8+t \rangle$

$\langle 4+t, 0, 8+t \rangle$

2. Question 50

$r_1(t) = \langle 1,0,0 \rangle + t\langle -3,1,0 \rangle$

$\langle 1-3t, s, 0 \rangle$

$r_2(t) = \langle 0,1,1 \rangle + t\langle 2,0,1 \rangle$

$\langle 2t, 1, 1+v \rangle$

$\langle \frac{2}{5}, 1, 0 \rangle$

$2t = 1-3t$

$5t = 1$

$t = \frac{1}{5}$

$s = 1$

$0 = 1+v$

$v = -1$

12.3 Homework

37.

$\langle -1, 2, 2 \rangle \quad \langle x, x, x \rangle$

$-1x + 2x + 2x = 0$

$3x = 0$

$x = 0$

$\langle 0, 0, 0 \rangle$

55.

$-2 + 0 + 0$

$\frac{-2}{\sqrt{5}} \langle 2, 0, 1 \rangle$

$\left\langle \frac{-4}{5}, 0, \frac{-2}{5} \right\rangle$

12.4 Homework

19. $W \times u + W \times v$

$\langle 0, -3, -1 \rangle + \langle -2, 1, -1 \rangle$

$\langle -2, -2, -2 \rangle$

$1 + 16 + 49$ 65

$\sqrt{66}$

30

i	j	k
3	1	1
-1	2	1

$(1-2)i - (3+1)j + (6+1)k$

$-1i - 4j + 7k$

$\left\langle \frac{-1}{\sqrt{66}}, \frac{-4}{\sqrt{66}}, \frac{7}{\sqrt{66}} \right\rangle$

$\left\langle \frac{1}{\sqrt{66}}, \frac{4}{\sqrt{66}}, \frac{7}{\sqrt{66}} \right\rangle$

125. Homework

18:

$\langle -4, 0, 1 \rangle$

$\langle -3, 0, 0 \rangle$

$$\begin{matrix} i & j & k \\ -4 & 0 & 1 \\ -3 & 0 & 0 \end{matrix}$$

$0\hat{i} - 3\hat{j} - 0k$

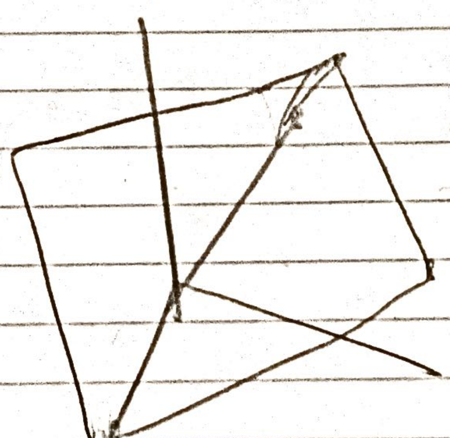
$0 - 3(x-1) - 0 =$

$-3x + 3$

$-3x = -3$

$x = 1$

3:

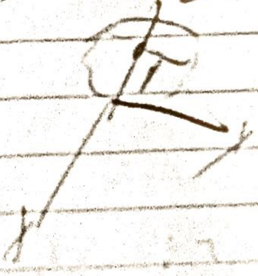


13.1 Homework

3. $r(2) = 2, 4, 5^{-1} \rangle$

$r(-1) = \langle -1, 1, 2^{-1} \rangle$

2 14 $\sin t\, i + (4 + \cos t\, j)$ $\sin^2 (4 + \cos^2 t)$ $\sin^2 + \cos^2 = 1$



Center $(0, 0, 4)$

xz plane

Radius 1

13.2 Homework

43. $\langle t^3, 2t^3, -\frac{\sin(3t)}{3} \rangle \Big|_0^1$

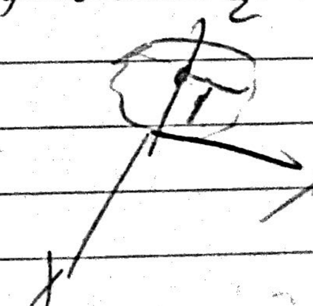
$\langle 1, 2, -\frac{\sin(3)}{3} \rangle$ - zero vector

13.1 Homework

3. $r(2) = \langle 2, 4, 5^{-1} \rangle$

$r(-1) = \langle -1, 1, 2^{-1} \rangle$

2 19 $\sin t\, i + (4 + \cos t\, j)$ $\sin^2 (4 + \cos^2 t)$ $\sin^2 + \cos^2 = 1$



Center $(0, 0, 4)$

xz plane

Radius 1

13.2 Homeworks

43. $\left\langle t^3, 2t^2, -\frac{\sin(3t)}{3} \right\rangle \Big|_0^1$

$\left\langle 1, 2, -\frac{\sin(3)}{3} \right\rangle$ - zerovero

5)

$\int r''(t)$

$r(t) = i + tj + (8t^2)$

$r'(t) = j + 16tk$

$j + \frac{t^2}{2} j + 8t^2 k$

$c_1 i + c_2 j + 16t k$

$\int r'(t)$

$c_1 j + tj + 8t^2 k$

13-4 Homework

12. $r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r'(1) = \langle 1, 2, 3 \rangle$ $\quad 1+4+9$

$\sqrt{14}$

10. $r'(t) = \langle \frac{2}{\sqrt{t}}, \frac{1}{t}, 2 \rangle$

$\int_1^t \sqrt{\frac{4}{t} + \frac{1}{t^2} + 4}$

$\int_1^t \sqrt{\frac{4t}{t^2} + \frac{1}{t^2} + \frac{4t^2}{t^2}}$

$\int_1^t \sqrt{\frac{4t^2+4t+1}{t^2}}$ $\quad 1-2t+t^{-2}$

$\int_1^t \sqrt{\frac{(2t+1)^2}{t^2}}$

$\int_1^t \frac{2t+1}{t} dt = \int_1^t 2 + t^{-1} dt$

$2t - t^{-2} \Big|_1^t$

$2 - 1 - 2t + t^{-2}$

13.5 Homework

$r(t) = \langle (1+s^2)^{-1},$

6. $r'(t) = \left\langle \frac{-2s}{(1+s^2)^2}, \frac{1+s^2 - 2s}{(1+s^2)^2} \right\rangle$

$r''(t) = \left\langle \frac{-2(1+s^2)^2 + 8(1+s^2)s}{(1+s^2)^4}, \frac{(-2s)(1+s^2)^2 + 8(1+s^2)s^2}{(1+s^2)^4} \right\rangle$

16. $v(t) \int a(t)$

$\langle e^t, t^2, \frac{t^2}{2} + t \rangle$

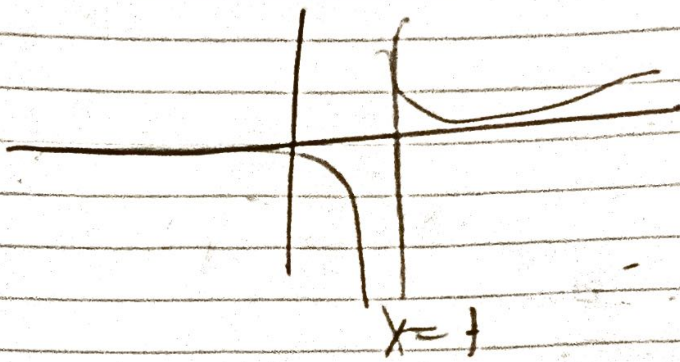
$v(t) = \langle e^t, t^2, \frac{t^2}{2} + t + 1 \rangle$ $a(t) = \langle e^t + 1, t^2 + 1, \frac{t^2}{2} + t + 1 \rangle$

$r(t) = \int v(t)$ $2 = 1 + C$

$= \langle e^t + 1, t^2 + 1, \frac{t^2}{2} + t + 1 \rangle$

14.1 Homework

5



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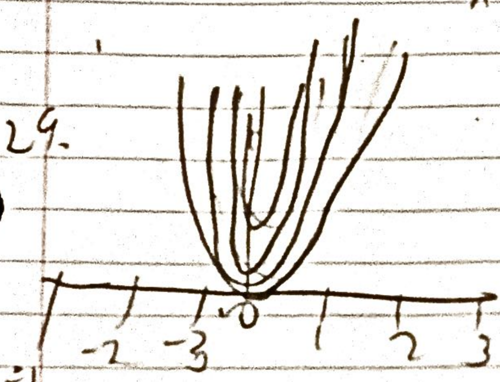


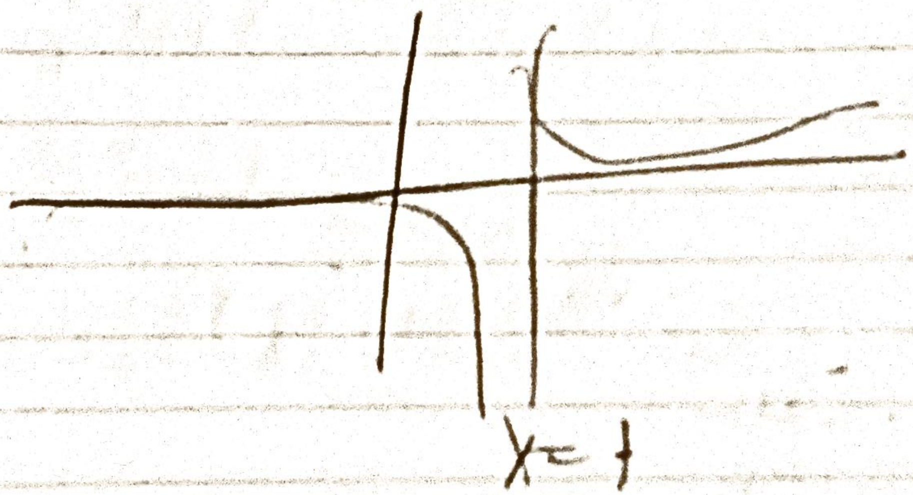
14.2

33 $\lim\limits_{(x,y)\to(1,3)} e^{x-y}\ln(\dots)$ $e^{?}\ln(y)$

37. $\tan(-0) = \tan^{-1}(\frac{1}{0}) = 0$

14.1 Homework

8



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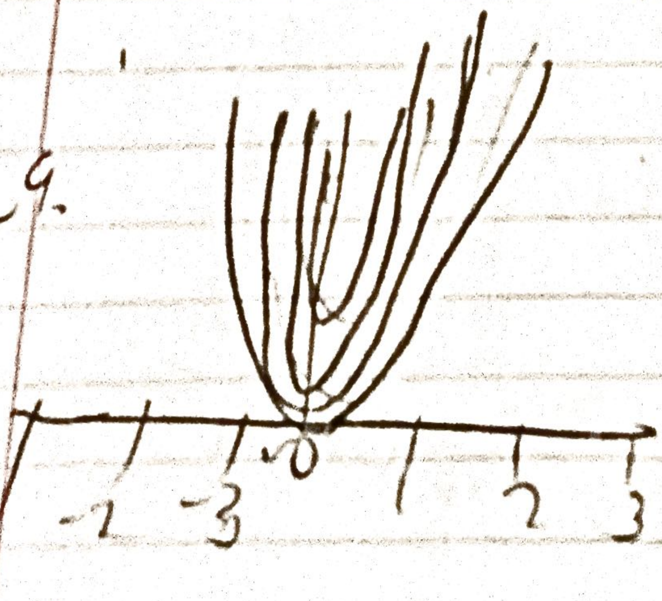

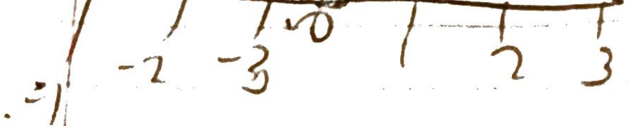

14.2

33. $\lim_{(x,y)\to(1,3)} e^{x-y}\ln(-x-y))$ $e^{y}\ln(y)$

37. $\tan(-0)$ $\tan^{-1}(\frac{1}{0})$ $=$ 0

14.3 Homework

29. $\frac{\partial}{\partial y} \rightarrow ye^{xy}$

$\frac{\partial}{\partial x} = xe^{xy}$

43. $g_u(u,v) = \ln(u+v) + \frac{u}{u+v}$

$\ln(3) + \frac{1}{3}$

1.

14.4

1a $f_x(1,1) = \frac{8x}{4x^2-y^2} = \frac{8}{3}$

$f_y(1,1) = \frac{-2y}{4x^2-y^2} = \frac{-2}{3}$

$\frac{8}{3}x - \frac{2}{3}y - 2$

$\frac{8}{3}(x-1) - \frac{2}{3}(y-1)$

$\frac{8}{3}x - \frac{8}{3} - \frac{2}{3}y + \frac{2}{3}$

28. $\frac{8}{\sqrt{2\times 2}} = \frac{8}{2} = 4$

14.5 Homework

15 $f(r(t)) = t^2 - (t^2)(t^2 - 4t)$

$= t^2 - t^4 + 4t^3$

$f'(r(t)) = 2t - 4t^3 + 12t^2$

$8 - 4(4^3) + 12(16)$

-56

#3.

$(1, -1, 1)$

$F_x = 2xy - y^2 = -2 - 1 = -3$

$F_y = x^2 - 2yx = 1 + 2 = 3$

$F_z = 9z^2 = 9$

$\langle -3, 3, 9 \rangle$

14.6 Homework

6. $\frac{\partial R}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial u}$

$\frac{\partial R}{\partial u} = 50(3(u^2) + 4(uv))^4\, 2u + 20(3u^2 + 4uv)^4(v)$

$\frac{\partial R}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial v}$

$15(3u^2 + 4uv)^4(0) + 20(3u^2 + 4uv)^4 u$

$20(3u^2 + 4uv)^4 u$

13. $\frac{\partial g}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial g}{\partial y}\frac{\partial x}{\partial \theta}$ $\left(2\sqrt{2}, \frac{\pi}{4}\right)$

$-r(r\cos\theta + (r\sin\theta))^{-2} + r(r\cos\theta + (r\sin\theta))^{-2}$ $\frac{4}{6}$

$= -2\left(2\sqrt{2}\,\frac{\sqrt{2}}{2} + 2\sqrt{2}\,\frac{\sqrt{2}}{2}\right)^{-2} + (2\sqrt{2}\,\frac{\sqrt{2}}{2} + 2\sqrt{2}\,\frac{\sqrt{2}}{2})^{-2}$

$= -2(2+4)^{-2} + (2+4)^{-2} = \frac{-1}{6}$

14.7 Homework

21. $f_x(x,y) = 1 - \frac{1}{x+y}$ $\quad 1 = \frac{1}{x+y}$

$f_y(x,y) = -2y - \frac{1}{x+y}$ $\quad x+y=1$

$y = 1-x$

$-2y - 1 = 0$

$y = -\frac{1}{2}$

$\left(\frac{3}{2}, -\frac{1}{2}\right)$ - Saddle point

$f_{xx} = (x+y)^{-2}$ $\quad f_{xy} = (x+y^{-2})$

$f_{yy} = -2 + (x+y)^{-2}$

$\frac{3}{2} + 4$

$\frac{3}{2} + \frac{8}{2}$

$(1)(-2+1)$

$-1 - \frac{11}{2} < 0$

0 ,
0 5

31.

$f(0,0) = 1$
$f(0,5) = \frac{1}{26}$
$f(3,5) = \frac{1}{35}$
$f(0,3) = \frac{1}{10}$

global max: 1
global min $\frac{1}{35}$