Second Chance Club Materials Daniel Lim

Questions From Previous Finals

12.1 : N/A

12.2 : N/A

12.3 : N/A

12.4 : N/A

12.5

$r(t) = \langle 1,1,1\rangle + t\langle 1,-1,0\rangle$ point $= (1,0,2)$

$r(t) = \langle 1+t, 1-t, 1\rangle$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$1(x-1)+1(y-0)+1(z-2)=0 \Rightarrow \boxed{x+y+z=3}$

My Problem

Find eq. that passes through points $(1,1,1)$ $(2,0,2)$ $(2,2,0)$

$P=(1,1,1)$ $Q=(2,0,2)$ $R=(2,2,0)$ $PQ=\langle 1,-1,1\rangle$ $PR=\langle 1,1,-1\rangle$

$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \langle 0,2,2\rangle$ Using P as the reference point

$0(x-1)+2(y-1)+2(z-1)=0$ $\boxed{y+z=2}$

13.1: N/A

13.2

$a(t) = i + j \rightarrow \int a(t) = \int i + j\,dt \rightarrow v(t) = (t+c)i + (t+c)j$

At $t=0$, $v = i+j$ $\rightarrow \boxed{v(t) = (t+1)i + (t-1)j}$

$\int v(t) = \int (t+1)i + (t-1)j\,dt = \left(\frac{t^2}{2}+t\right)i + \left(\frac{t^2}{2}-t\right)j + c$

At $t=0$, $r=k$ $\rightarrow \boxed{r(t) = \left(\frac{t^2}{2}+t\right)i + \left(\frac{t^2}{2}-t\right)j + k}$

My Problem

Find $r(t)$ if $r'(t) = \langle t, 3t^2, 2t^3\rangle$ and $r(1) = \langle 2,1,4\rangle$

$r(t) = \int r'(t) = \frac{t^2}{2}i + t^3 j + \frac{1}{2}t^4 + C$

$\frac{1}{2}i + 1j + \frac{1}{2}k + C = 2i + 1j + 4k \rightarrow C = \frac{3}{2}i + \frac{7}{2}k$

$\boxed{r(t) = \left(\frac{t^2}{2} - \frac{3}{2}\right)i + t^3 j + \left(\frac{1}{2}t^4 - \frac{7}{2}\right)k}$

13.3

a) $r(t) = \langle e^t \cos t,\ e^t \sin t,\ e^t \rangle,\quad 0 \le t \le \pi$

$v(t) = \frac{dr}{dt} = \langle \frac{dr}{dt}(e^t \cos t),\ \frac{dr}{dt}(e^t \sin t),\ \frac{dr}{dt}(e^t) \rangle$

$\boxed{v(t) = \langle e^t \cos t - e^t \sin t,\ e^t \sin t + e^t \cos t,\ e^t \rangle}$

b) $L = \int_0^\pi \|r'(t)\|\,dt \rightarrow \int_0^\pi \|v(t)\|\,dt$

$\|v(t)\| = \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t + e^{2t}} = \sqrt{2e^{2t} + e^{2t}} = \sqrt{3e^{2t}}$

$L = \int_0^\pi \sqrt{3e^{2t}}\,dt = \int_0^\pi \sqrt{3}\,e^t\,dt = \sqrt{3}\,[e^t]_0^\pi = \boxed{\sqrt{3}(e^\pi - 1)}$

My Problem

Find arc length of $r(t) = \langle \frac{1}{2}t^2,\ t^2,\ 3t^2 \rangle$ for $0 \le t \le 2$

$r'(t) = \langle t, 2t, 6t \rangle \quad \|r'(t)\| = \sqrt{t^2 + 4t^2 + 36t^2} = \sqrt{41t^2}$

$L = \int_0^2 \sqrt{41t^2}\,dt = \sqrt{41}\int_0^2 t\,dt = \sqrt{41}\,[\frac{t^2}{2}]_0^2 = \boxed{2\sqrt{41}}$

13.4

$r(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle \quad r'(t) = \langle 1, 2t, 2t^2 \rangle \quad r''(t) = \langle 0, 2, 4t \rangle$

$r'(t) \times r''(t) = \langle 4t^2, -4t, 2 \rangle \quad \|r'(t) \times r''(t)\| = 2\sqrt{(2t^2+1)^2}$

$\|r'(t)\| = \sqrt{1 + 4t^2 + 4t^4} \quad K = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \boxed{\frac{2}{9}}$

My Problem

Find the curvature for $r(t) = \langle 3t, 4t^2, 7t \rangle$

$r'(t) = \langle 3, 8t, 7 \rangle \quad r''(t) = \langle 0, 8, 0 \rangle$

$r'(t) \times r''(t) = (-56, 0, 24) \rightarrow \|r'(t) \times r''(t)\| = 8\sqrt{58}$

$\|r'(t)\| = \sqrt{64t^2 + 58} \quad \boxed{K = 8\sqrt{58} / (\sqrt{64t^2+58})^3}$

13.5: N/A

14.1: There was no link to the lecture notes

14.2

$\lim_{(x,y,z)\to(1,1,1)} e^{-xy}\sin(\pi z/2) \rightarrow e^{-(1)(1)}\sin(\pi(1)/2) = \boxed{\frac{1}{e}}$

My Problem

Compute the limit $\lim_{(x,y,z)\to(1,1,1)} \frac{x+y+z}{x+y-z}$

$\lim_{(x,y,z)\to(1,1,1)} \frac{x+y+z}{x+y-z} = \frac{1+1+1}{1+1-1} = \boxed{\frac{3}{2}}$

14.3 : N/A

14.4

$z = e^{2x-3y}$ at point $(3,2,1)$

$f_x = 2e^{2x-3y}$ $\quad f_y = 3e^{2x-3y}$

$f(3,2) = 2e^0 = 2$ $\quad f(3,2) = -3e^0 = -3$

$z - 1 = 2(x-3) - 3(y-2) \rightarrow \boxed{z = 2x - 3y + 1}$

My Problem

Find equation of tangent plane to surface $z = 3x^2y + 2x$ at $(1,2,8)$

$f_x = 6xy + 2$ $\quad f_y = 3x^2$

$f(1,2) = 14$ $\quad f(1,2) = 3$

$z - 8 = 14(x-1) + 3(y-2) \Rightarrow \boxed{z = 14x + 3y - 12}$

14.5

a) $\nabla f = f_x + f_y + f_z$

$f_x = -2x$ $\quad f_y = 2y$ $\quad f_z = 2z$ $\quad \boxed{\nabla f = \langle -2x, 2y, 2z \rangle}$

b) $\nabla f = \langle -2x, 2y, 2z \rangle \cdot \nabla f(1,1,1) = \langle -2, 2, 2 \rangle$

$-2(x-1) + 2(y-1) + 2(z-1) = 0 \rightarrow \boxed{z = x - y + 1}$

c) $\nabla f = \langle -2x, 2y, 2z \rangle$ $\quad \vec{u} = \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$\langle -2x, 2y, 2z \rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle = \frac{1}{3}(-2x + 4y + 4z) = \boxed{2}$

My Problem

Compute directional of $f(x,y,z) = 2x^2 + y^2 - 3z^2$ at point $(1,1,1)$ in $\langle 2,1,1 \rangle$

$\nabla f = \langle 4x, 2y, -6z \rangle$ $\quad \hat{u} = \langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$

$\langle 4x, 2y, -6z \rangle \cdot \langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle = \boxed{4\sqrt{\tfrac{2}{3}} - 2\sqrt{2}/\sqrt{3}}$

14.6

$f(x,y) = x^3 + 2xy + y^3$

$\frac{\partial f}{\partial r} = 3x^2 + 2y + 3y^2 + 2x \quad x = r - s \quad y = r + s$

$= 3(r-s)^2 + 2(r+s) + 3(r+s)^2 + 2(r-s)$

$\frac{\partial f}{\partial s} = -3x^2 - 2(y) + 3(y)^2 + 2x$

$= -3(r-s)^2 - 2(r+s) + 3(r+s)^2 + 2(r-s)$

$\frac{\partial z}{\partial x} = (5yz - \cos(x+2y+3z)) / (3\cos(x+2y+3z) - 5xy)$

$\frac{\partial z}{\partial y} = (5xz - 2\cos(x+2y+3z)) / (3\cos(x+2y+3z) - 5xy)$

My Problem

Use chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ $\quad z = 2x^2 + 3xy^2 + y^3 \quad x = s + t \quad y = st$

$\frac{\partial z}{\partial x} = 4x + 3y^2 \quad \frac{\partial z}{\partial y} = 6xy + 3y^2 \quad \frac{\partial x}{\partial s} = 1 \quad \frac{\partial x}{\partial t} = 1 \quad \frac{\partial y}{\partial s} = t \quad \frac{\partial y}{\partial t} = s$

$\frac{\partial z}{\partial s} = 4x + 3y^2 + 6xyt + 3ty^2 \rightarrow 4(s+t) + 3(st)^2 + 6(s+t)(st)t + 3t(st)^2$

$\frac{\partial z}{\partial t} = 4x + 3y^2 + 6xys + 3sy^2 \rightarrow 4(s+t) + 3(st)^2 + 6(s+t)(st)s + 3s(st)^2$

14.7

1. $f(x,y) = 4x^2 + y^2 + 2x^2y - 1$

$f_x = 8x + 4xy \quad f_y = 2y + 2x^2 \quad f_{xx} = 8 + 4y \quad f_{yy} = 2 \quad f_{xy} = 8 + 4y$

$D = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

$8x + 4xy = 0 \qquad 2y + 2x^2 = 0 \rightarrow y = -x^2$

$8x - 4x^3 = 0 \rightarrow x(8 - 4x^2) = 0 \quad x = 0, \sqrt{2}, -\sqrt{2}$

$D(0,0) > 0 \quad f_{xx}(0,0) > 0 \rightarrow$ rel. min at $(0,0) = -1$

$D(\sqrt{2}, -2) < 0$ so $(\sqrt{2}, -2)$ and $(-\sqrt{2}, -2) \rightarrow$ saddle points

2. $f(x,y) = 6y^2 - 2y^3 + 3x^2 + 6xy$

$f_x = 6x + 6y \quad f_y = 12y - 6y^2 + 6x \quad f_{xx} = 6 \quad f_{yy} = 12 - 12y \quad f_{xy} = 6$

$0 = 6x + 6y \rightarrow -x = y \quad 0 = 12y - 6y^2 + 6x$

$0 = 12y - 6y^2 + 6y \rightarrow 0 = -6y^2 + 18y \rightarrow 0 = y(-6y + 18)$

$D(0,0) > 0 \quad f_{xx}(0,0) > 0 \rightarrow$ rel min 0

My Problem

Find local min max saddle of $f(x,y) = 3x^2 + y^3 + 4xy$

$f_x = 6x + 4y \quad f_y = 3y^2 + 4x \quad f_{xx} = 6 \quad f_{yy} = 6y \quad f_{xy} = 4$

$D = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

$0 = 6x + 4y \quad 0 = 3y^2 + 4x \rightarrow -3y^2 = 4x \rightarrow x = -\frac{3}{4}y^2$

$0 = -\frac{9}{2}y^2 + 4y \rightarrow 0 = y(-\frac{9}{2}y + 4) \rightarrow y = 0, \frac{8}{9}$

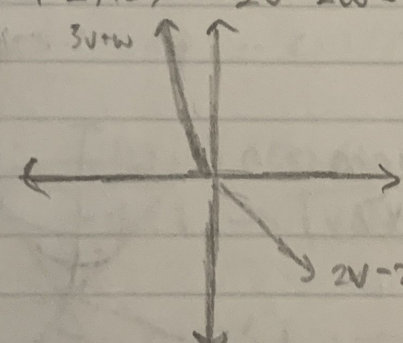
$D(0,0) = 6(0) - (4)^2 = -16 \rightarrow$ saddle point

$D(-\frac{16}{27}, \frac{8}{9}) = 6(6(\frac{8}{9})) - 16 > 0$ and $f_{xx} = 6 > 0$ relative minimum

12.1 2 Exercises

23. Sketch $V = \langle 0,2 \rangle$, $W = \langle -2,4 \rangle$, $3v+w$, $2v-2w$

$3v+w = \langle -2,10 \rangle \quad 2v-2w = \langle 4,-4 \rangle$



33. $A = (1,1)$, $B = (3,4)$, $P = (1,1)$, $Q = (7,10)$

$\overrightarrow{AB} = \langle -8,1 \rangle \quad \overrightarrow{PQ} = \langle 6,9 \rangle$, They're parallel, pointing to the same side

12.2 2 Exercises

21. $\frac{1}{2}\langle 4,-2,8 \rangle - \frac{1}{3}\langle 12,3,3 \rangle \rightarrow \langle 2,-1,4 \rangle - \langle 4,1,1 \rangle = \boxed{\langle -2,-2,3 \rangle}$

29. e_w where $W = \langle 4,-2,-1 \rangle \quad \sqrt{4^2 + (-2)^2 + 1} = \sqrt{21}$

$e_w = \langle 4,-2,-1 \rangle / \sqrt{21} \rightarrow \boxed{\langle \frac{4}{\sqrt{21}}, -\frac{2}{\sqrt{21}}, -\frac{1}{\sqrt{21}} \rangle}$

12.3 2 Exercises

19. $\cos\theta = \frac{a \cdot b}{\|a\| \cdot \|b\|} = \frac{\langle 0,3,1 \rangle \cdot \langle 4,0,0 \rangle}{\sqrt{10} \cdot 4} \rightarrow \cos^{-1}(") = \boxed{0}$

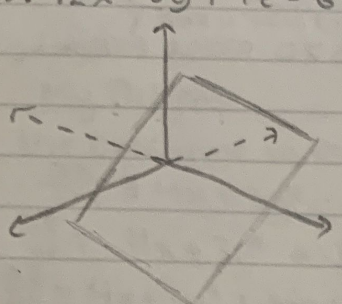
12.4 2 Exercises

17. $v \times u$ $\quad u \times v = \langle 1,1,0\rangle$ $\quad v \times u$ is opposite so $\rightarrow \boxed{\langle -1,-1,0\rangle}$

19. $w \times (u+v) = (w \times u) + (w \times v) = \langle 0,-3,-1\rangle + \langle -2,1,-1\rangle = \boxed{\langle -2,-2,-2\rangle}$

12.5 2 Exercises

33. $12x - 6y + 4z = 6$



39. $x+y+z=14$ $\quad r(t) = \langle 1,1,0\rangle + t\langle 0,2,4\rangle$

$x = 1 + t(0) = 1$

$y = 1 + 2t \rightarrow 1 + 2(2) = 5$

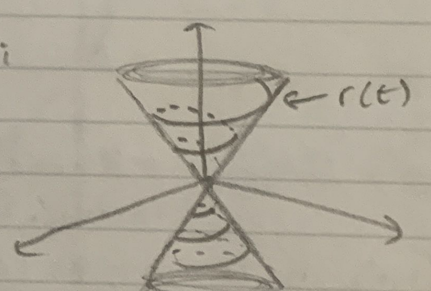
$z = 4t \rightarrow 4(2) \Rightarrow 8$

Solve for $t \rightarrow t = 2$ $\quad \boxed{\langle 1,5,8\rangle}$

13.1 2 Exercises

15. C and i, B and iii, A and ii

21. $r(t) = \langle t\cos t, t\sin t, t\rangle$



13.2 2 Exercises

29. $r(t) = \langle t^2, t^4\rangle, \; t = -2$ $\quad r(2) = \langle 4, 16\rangle$

$r'(t) = \langle 2t, 4t^3\rangle$ $\quad r'(2) = \langle -4, -32\rangle \rightarrow \boxed{\langle 4-4t, 16-32t\rangle}$

43. $\int_0^1 \langle 2t, 4t, -\cos 3t\rangle \, dt$

$\langle t^2, 2t^2, -\frac{1}{3}\sin 3t\rangle \Big|_0^1 \rightarrow \boxed{\langle 1, 2, -\frac{1}{3}\sin 3\rangle}$

13.3 2 Exercises

51. $r(t) = \langle \cos t, \sin t, \frac{2}{\sqrt{3}} t^{3/2}\rangle$ $\quad (0,0,0)$

$r'(t) = \langle -\sin t, \cos t, \frac{3}{\sqrt{3}} t^{1/2}\rangle$

$r(s) = \langle \cos\left((\frac{3}{2}s+1)^{2/3} - 1\right), \sin\left((\frac{3}{2}s+1)^{2/3} - 1\right), \frac{2}{3}\left((\frac{3}{2}s+1)^{2/3} - 1\right)^{3/2}\rangle$

33. $r(t) = \langle t^2, t^3 \rangle \quad r'(t) = \langle 2t, 3t^2 \rangle$

$S = \int_a^b \|r'(t)\| \, dt \quad \|r'(t)\| = \sqrt{4t^2 + 9t^4} = t\sqrt{4+9t^2}$

$r(s) = \left\langle \frac{1}{9}(27s+8)^{2/3} - \frac{4}{9}, \frac{1}{27}\left((27s+8)^{2/3} - 4\right)^{3/2} \right\rangle$

13.4 2 Exercises

15. $y' = e^t \quad y'' = e^t \quad K = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \boxed{\frac{e^3}{(3^6+1)^{3/2}}}$

31. $\langle t\cos t, \sin t \rangle \quad t = \pi \quad r'(t) = \langle \cos t - t\sin t, \cos t \rangle$

$r''(t) = \langle -\sin t - \sin t - t\cos t, -\sin t \rangle$

$K = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} \rightarrow k(\pi) = \boxed{\frac{\pi\sqrt{2}}{4}}$

13.5 2 Exercises

13. $a(t) = k, \; v(0) = i \quad v(t) = \int a(t)\,dt = tk + C$

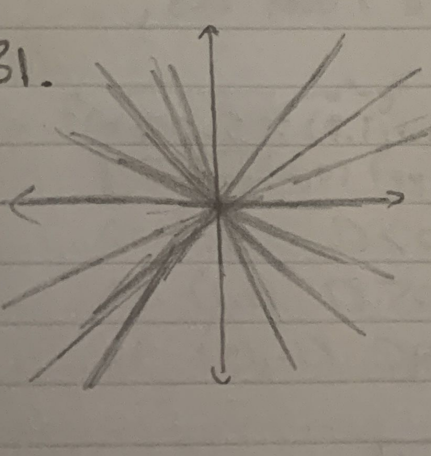
$i = tk + C \rightarrow C = i \rightarrow \boxed{v(t) = i + tk}$

33. $r(t) = \langle t, \cos t, \sin t \rangle \quad r' \langle 1, -\sin t, \cos t \rangle \quad r''(t) = \langle 0, -\cos t, -\sin t \rangle$

$a_T = v'(t) \rightarrow \boxed{0} \quad a_N = (v(t)^2)k(t) \rightarrow \boxed{1}$

14.1 2 Exercises

31.



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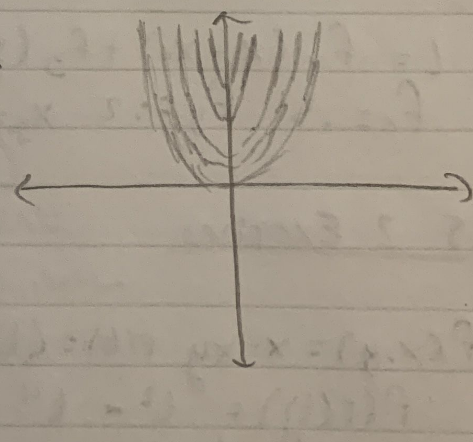

14.2 2 Exercises

33. $\lim_{(x,y)\to(1,-3)} e^{x-y}\ln(x-y) = e^{1+3}\ln(1+3) = \boxed{e^4 \ln 4}$

29. $\lim_{(x,y)\to(4,2)} = \frac{y-2}{\sqrt{x^2-4}} = \frac{2-2}{\sqrt{16-4}} = \boxed{0}$

14.3 2 Exercises

37. $W = xy^2z^3$ $\quad \frac{\partial w}{\partial x} = y^2z^3$ $\quad \frac{dw}{dy} = 2xyz^3$ $\quad \frac{dw}{dz} = 3xy^2z^2$

41. $f(x,y) = 3x^2y + 4x^3y^2 - 7xy^5$ $\quad f_x(1,2)$

$f_x = 6xy + 12x^2y^2 - 7y^5$

$f_x(1,2) = 6(1)(2) + 12(1)^2(2)^2 - 7(2)^5 = \boxed{-164}$

14.4 2 Exercises

19. $f(x,y,z) = z\sqrt{x+y}$ at $(8,4,5)$

$f_x = \frac{z}{\sqrt{x+y}} \quad f_x(8,4,5) = \frac{5}{2\sqrt{3}}$

$f_y = \frac{z}{\sqrt{x+y}} \rightarrow f_y(8,4,5) = \frac{5}{2\sqrt{3}}$

$f_z = \sqrt{x+y} \rightarrow f_z(8,4,5) = 2\sqrt{3}$

$L = f_x(x-8) + f_y(y-4) + f_z(z-5) = \boxed{\frac{5}{12}\sqrt{3}x + \frac{5}{12}\sqrt{3}y + 2\sqrt{3}z - 5\sqrt{3}}$

21. $L = f_x(x-x_0) + f_y(y-y_0)$

$f_x = .3 \quad f_y = -.2 \quad x_0 = 2 \quad y_0 = 4 \quad L(2.1, 3.8) = \boxed{5.07}$

14.5 2 Exercises

15. $f(x,y) = x - xy \quad r(t) = \langle t^2, t^2 - 4t\rangle \quad t = 4$

$f(r(t)) = t^2 - t^4 + 4t^3$

$\frac{d}{dt}(t^2 - t^4 + 4t^3)\big|_{t=4} = \boxed{-56}$

21. $f(x,y) = x^2 + y^3$ $v = \langle 4,3 \rangle$ $P = (1,2)$
$f_x = 2x$ $f_y = 3y^2$ $\vec{u} = \langle \frac{4}{5}, \frac{3}{5} \rangle$
$D_u = \frac{4}{5}(x^2) + \frac{3}{5}(y^3) \rightarrow D_u(1,2) = \boxed{8.8}$

14.6 2 Exercises

13. $\frac{\partial g}{\partial \theta} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial \theta}$ $x = r\cos\theta$ $y = r\sin\theta$
$\frac{\partial g}{\partial x} = \frac{1}{(x+y^2)^2}$ $\frac{\partial g}{\partial y} = \frac{2y}{(x+y^2)^2}$ $x_\theta = -r\sin\theta$ $y_\theta = r\cos\theta$
$\left(\frac{1}{(x+y^2)^2}\right)(-r\sin\theta) + \left(\frac{2y}{(x+y^2)^2}\right)(r\cos\theta) = \boxed{-1/6}$

28. $\frac{\partial w}{\partial z}$, $x^2w + w^3 + wz^2 + 3yz = 0$
$\partial w = x^2 + 3w^2 + z^2$ $\partial z = 2wz + 3y$ $\boxed{-\frac{2wz + 3y}{x^2 + 3w^2 + z^2}}$

14.7 2 Exercises

17. $f(x,y) = \sin(x+y) - \cos x$
$f_x = \cos(x+y) + \sin x$ $f_y = \cos(x+y)$ $f_{xx} = \cos x - \sin(x+y)$
$f_{yy} = -\sin(x+y)$ $f_{xy} = -\sin(x+y)$
$(j\pi, k\pi + \frac{\pi}{2})$ = critical points
j k even = $D < 0$ so saddle points = j odd k even
j k odd = $D > 0$ $f_{xx} < 0$ local max
j even k odd = $D > 0$ $f_{xx} > 0$ local min

15. $f(x,y) = xye^{-x^2-y^2}$
$D = f_{xx}(a,b)\, f_{yy}(a,b) - [D_{xy}(a,b)]^2$
at $(0,0)$ $D < 0$ = saddle point
at $(1,1)$ $D > 0$, $f_{xx} > 0$ local min
at $(-1,-1)$ $D > 0$, $f_{xx} > 0$ local min