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Second Chance Club

For exam 1.

handout 12.5

(problem from previous final)

Find an equation for the plane through the point $(1,0,2)$ that contains the line $r(t) = (1,1,1) + t(1,-1,0)$

Answer: $r(t) = (1,1,1) + (1,-1,0)t$

$= (t+1, 1-t, 1)$

B: when $t=0$, $r(0) = (1,1,1)$ $\quad A = (1,0,2)$

C: $t=1$, $r(1) = (2,0,1)$

$AB = (0,1,-1)$

$AC = (1,0,-1)$

$AB \times AC = (-1,-1,-1)$

$-1(x-1) - 1(y-0) - 1(z-2) = 0$

$-x + 1 - y - 2 + z = 0$

$-x - y - z = -3$

$x + y + z = 3$

(similar problem)

find an equation of the plane passing through the three points given: $P = (5,1,1)$ $Q = (1,1,2)$ $R = (2,1,1)$

$PQ = (-4,0,1)$

$PR = (-3,0,0)$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -4 & 0 & 1 \\ -3 & 0 & 0 \end{vmatrix}$$

$= 0i - j(-4\cdot 0 + 3) + 0k$

$= \cancel{0} -3j = (0,-3,0)$

$0(x-5) - 3(y-1) + 0(z-1) = 0$

$-3y + 3 = 0$

$3y = +3$

$y = 1$

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handout 13.2 (problem from previous final)

find the velocity and position vectors of a particle whose acceleration is $a(t) = i + j$. and time $t=0$, the velocity is $i-j$ and position is k.

Answer: $\because a(t) = i + j$

$$\int a(t)\,dt = v(t) = ti + tj + C$$

when $t=0$, $v(0) = i - j = C$

$$\therefore v(t) = ti + tj + i - j$$

$$= i(t+1) + j(t-1)$$

$$\int v(t)\,dt = r(t) = (\tfrac{1}{2}t^2 + t)i + (\tfrac{1}{2}t^2 - t)j + C$$

$r(0) = k = C$

$$\therefore r(t) = (\tfrac{1}{2}t^2 + t)i + (\tfrac{1}{2}t^2 - t)j + k$$

similar question:

find the velocity and ~~position~~ acceleration vectors of a particle whose position is $r(t) = \cos t\, i + 5\sin t\, j + 5k$

Answer: $r'(t) = v(t) = -\sin t\, i + 5\cos t\, j$

$a(t) = r''(t) = v'(t) = -\cos t\, i - 5\sin t\, j$

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no 13.3

(Prob from previous final:)

suppose that the position of a certain particle is given by

$r(t) = (e^t \cos t, e^t \sin t, e^t) \quad 0 \le t \le \pi$

(a) find the velocity of the particle as a function of time t.

$r'(t) = v(t) = (e^t \cos t + (-e^t \sin t), e^t(\sin t + \cos t), e^t)$

(b) find the length of the arc traversed by the moving particle for $0 \le t < \pi$

$\therefore \|r'(t)\| = \sqrt{(e^t(\cos t - \sin t))^2 + (e^t(\sin t + \cos t))^2 + (e^t)^2}$

$\int_0^\pi \|r'(t)\| dt = -\sqrt{3} + \sqrt{3} \cdot e^\pi$

$= \sqrt{3}(e^\pi - 1)$

(Similar prob:)

find the acceleration of the position vector $r(t) = (3t, 4t-3, 6t+1)$, $0 \le t \le 3$ find the length of the curve over the given region.

Ans: $a(t) = r''(t) = (0,0,0)$ $\qquad r'(t) = (3,4,6)$

arc length => $\|r'(t)\| = \sqrt{3^2 + 4^2 + 6^2}$

$= \sqrt{61}.$

$\int_0^3 \sqrt{61}\, dt$

$= 3\sqrt{61}$

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■ ho13.4

(Prob from previous final)

find the curvature of the curve $r(t) = (t, t^2, \frac{2}{3}t^3)$ at the point $(1, 1, \frac{2}{3})$

Ans: $r'(t) = (1, 2t, 2t^2)$

$r''(t) = (0, 2, 4t)$

$$r'(t) \times r''(t) = \begin{vmatrix} 1 & 2t & 2t^2 \\ 0 & 2 & 4t \end{vmatrix}$$

$$= (8t^2 - 4t^2)i - 4tj + 2k$$

$$= (4t^2, -4t, 2)$$

$$\|r'(t) \times r''(t)\| = \sqrt{16t^4 + 16t^2 + 4}$$

$\because t = 1$

$$= \sqrt{16 + 16 + 4}$$

$$= 6$$

$$|r'(t)| = \sqrt{1 + 4t^2 + 4t^4} = \sqrt{9} = 3$$

$$k(t) = \frac{6}{3^3} = \frac{6}{27} = \frac{2}{9}$$

(Similar question)

find the curvature of the curve $r(t) = (\cos t, \sin t, t$

$t = \frac{\pi}{2}$.

Ans: $r'(t) = (-\sin t, \cos t, 2t) = (-1, 0, \pi)$

$r''(t) = (-\cos t, -\sin t, 2) = (0, -1, 2)$

$$r'(t) \times r''(t) = \begin{vmatrix} -1 & 0 & \pi \\ 0 & -1 & 2 \end{vmatrix} = \pi i - (-2)j + (1)k$$

$$= (\pi, 2, 1)$$

$$\|r'(t) \times r''(t)\| = \sqrt{\pi^2 + 2^2 + 1^2} = \sqrt{5 + \pi^2}$$

$$|r'(t)| = \sqrt{(-1)^2 + \pi^2} = \sqrt{1 + \pi^2}$$

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$$k(t) = \frac{\sqrt{5 + \pi^2}}{(1 + \pi^2) \cdot \sqrt{1 + \pi^2}} = 0.1076$$

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no 14.2

compute the limit $\lim_{(x,y,z)\to(1,1,1)} e^{-xy}\cdot\sin(\frac{\pi z}{2})$ or prove it DNE.

Ans:

$$\lim_{(x,y,z)\to(1,1,1)} e^{-xy}\cdot\sin(\frac{\pi z}{2})$$

$$= e^{-1}\cdot\sin\frac{\pi}{2}$$

$$= e^{-1}\cdot 1 = e^{-1}$$

$\therefore$ it exist and the limit should be e^{-1}.

(similar prob).

find $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}}$ or DNE.

Ans: $\lim_{(x,y)\to(3,4)} \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{5}$

no 14.4

(prob from previous final)

Find an equation of the tangent plane to the surface.

$z = e^{2x-3y}$ at the point $(3,2,1)$

Ans: $f_x = 2\cdot e^{2x-3y} = 2\cdot e^0 = 2$

$f_y = -3\cdot e^{2x-3y} = -3$

$$z-1 = (2)(x-3)+(-3)(y-2)$$

$$z = 2x-6-3y+6+1$$

$$= 2x-3y+1$$

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(Similar prob) find an equation of the tangent plane at the given point.

$f(x,y) = \ln(4x^2 - y^2)$ $(1,1)$

Ans: $f_x = 8x \cdot \frac{1}{4x^2-y^2} = 8 \cdot \frac{1}{3} = \frac{8}{3}$

$f_y = -2y \cdot \frac{1}{4x^2-y^2} = -2 \cdot \frac{1}{3} = -\frac{2}{3}$

$z = \frac{8}{3}(x-1) + (-\frac{2}{3})(y-1)$

$= \frac{8}{3}x - \frac{2}{3}y + (\frac{2}{3} - \frac{8}{3}) = \frac{8}{3}x - \frac{2}{3}y - \frac{6}{3}$

$= \frac{8}{3}x - \frac{2}{3}y - 2$

no 14.5

let $f(x,y,z) = -x^2 + y^2 + z^2 - 1$

(a) compute ∇f.

$\nabla f = (-2x, 2y, 2z)$

(b) find a normal to the level surface $f(x,y,z) = 0$, at the point $(1,1,1)$ and give an equation for the tangent plane to that surface at that point.

$\nabla f = (-2, 2, 2)$ $f_x = -2$, $f_y = 2$, $f_z = 2$

$\cancel{z - 2 = -2(x-1) + 2(y-1)}$, $f(1,1,1) = 0$

$\cancel{z = -2x + 2 + 2y - 2 + 2}$ $(-1,1,1)$

$\cancel{z = -2x + 2y + 2}$

$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle -2, 2, 2 \rangle = (x-1)\cdot(-2) + 2(y-1) + 2(z-1)$

$-2x + 2y + 2z = 2$

$-x + y + z = 1$

$z = \cancel{x + y}\ x - y + 1$

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(c) compute the directional derivative of $f(x,y,z)$ at the point $(1,1,1)$ in the direction $(1,2,2)$

$\because \nabla f = (-2,2,2)$

~~$\|\nabla f\| = \sqrt{4+4+4} =$~~

$\|(1,2,2)\| = \sqrt{1+4+4} = 3$

$u = \frac{1}{3}\cdot(1,2,2) = (\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$

$\nabla f \cdot u = (-2,2,2)\cdot(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$

$= -\frac{2}{3} + \frac{4}{3} + \frac{4}{3} = 2$

$\therefore$ the directional derivative is 2.

(similar prob)

calculate the ∇f and the directional derivatives in the direction of v at the given point,

$f(x,y) = x^2y^3, \quad v = (1,1) \quad P = (\frac{1}{6}, 3)$

Ans: $\nabla f = (2xy^3, 3x^2y^2)$

$\|(1,1)\| = \sqrt{2}$

$u = \frac{1}{\sqrt{2}}\cdot(1,1) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$\nabla f = (2\cdot\frac{1}{6}\cdot 3^3, 3\cdot\frac{1}{6}^2\cdot 3^2) = (9, \frac{3}{4})$

$\nabla f \cdot u = (9, \frac{3}{4})\cdot(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$= \frac{9\sqrt{2}}{2} + \frac{3\sqrt{2}}{8} = \frac{36\sqrt{2}+3\sqrt{2}}{8}$

$= \frac{39\sqrt{2}}{8}$

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hO 14.6

Q1: find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as functions of r and s, if $f(x,y)=x^3+2xy+y^3$ and the variables are related by $x=r-s$, $y=r+s$

Ans: $\frac{\partial f}{\partial r}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r}$

$=(3x^2+2y)\cdot 1+(2x+3y^2)\cdot 1$

$=3x^2+2y+2x+3y^2$

$=3(r-s)^2+2(r+s)+2(r-s)+3(r+s)^2$

$\frac{\partial f}{\partial s}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}=(3x^2+2y)\cdot(-1)+(2x+3y^2)\cdot 1$

$=-3(r-s)^2+(-2(r+s))+3(r+s)^2+2(r-s)$

Q2: find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ if $\sin(x+2y+3z)=5xyz+1$

Ans:

$\frac{\partial z}{\partial x}$:

$\cos(x+2y+3z)\cdot(1+3\frac{\partial z}{\partial x})=5yz+5xy\cdot\frac{\partial z}{\partial x}$

$\cos(x+2y+3z)+3\cos(x+2y+3z)\cdot\frac{\partial z}{\partial x}=5yz+5xy\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial x}(3\cos(x+2y+3z)-5xy)=5yz-\cos(x+2y+3z)$

$\frac{\partial z}{\partial x}=\frac{5yz-\cos(x+2y+3z)}{3\cos(x+2y+3z)-5xy}$.

$\frac{\partial z}{\partial y}$:

$(2+3\frac{\partial z}{\partial y})\cos(x+2y+3z)=5xz+5xy\cdot\frac{\partial z}{\partial y}$

$\frac{\partial z}{\partial y}(3\cos(x+2y+3z)-5xy)=5xz-2\cos(x+2y+3z)$

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$$\frac{dz}{dy} = \frac{5xz - 2\cos(x+2y+3z)}{3\cos(x+2y+3z) - 5xy}$$

(similar prob)

find $\frac{dF}{dr}$ and $\frac{dF}{ds}$ from $F(u,v) = e^{u+v}$, $u = r^2$
$v = rs$

Ans: $\frac{dF}{dr} = \frac{dF}{du} \cdot \frac{du}{dr} + \frac{dF}{dv} \cdot \frac{dv}{dr}$

$= e^{u+v} \cdot 2r + e^{u+v} \cdot s$

$= e^{r^2+rs} \cdot \cancel{2(rs)}\,2r + e^{r^2+rs} \cdot s$

$= e^{r^2+rs} \cdot (2r+s)$

$\frac{dF}{ds} = e^{u+v} \cdot 0 + e^{u+v} \cdot r$

$= e^{r^2+rs} \cdot r$

HO14.7 (prob from previous final)

Find the local maximum and minimum points, the local maximum and minimum values and saddle points of function

$$f(x,y) = 4x^2 + y^2 + 2x^2y - 1$$

Ans: $f_x = 8x + 4xy$ $\quad f_y = 2y + 2x^2$

$f_{xx} = 8 + 4y$ $\quad f_{xy} = 4x$ $\quad f_{yy} = 2$

$8x + 4xy = 0$ $\quad 4x(2+y) = 0$ $\quad x = 0$ or -2

$2y + 2x^2 = 0$ $\quad 2(y + x^2) = 0$ $\quad y = -x^2 = 0$ or -4.

$D = (8+4y) \cdot (2) - (4x)^2 = 16 + 8y - 16x^2 = 16$

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~~$D = 16 + 8y - 16x^2$~~

=

$D > 0$, $f_{xx} = 8 > 0$

$\therefore$ $(0,0)$ is local mini point, the value is -1.

$(\sqrt{2}, -2)$ is saddle point.

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2 of hw questions in each section

Exercise 12.1

Q17. $(-\frac{1}{2}, \frac{5}{3}) + (3, \frac{10}{3})$

$= (-\frac{1}{2}+3, \frac{5}{3}+\frac{10}{3})$

$= (\frac{5}{2}, \frac{15}{3})$

$= (\frac{5}{2}, 5)$

Q50. Find a vector v satify

$3v + (5, 20) = (11, 17)$

$11 = 5 + 3v_x$

$3v_x = 6$

$v_x = 2$

$17 = 20 + 3v_y$

$3v_y = -3$

$v_y = -1$

$\therefore (2, -1)$

Exercise 12.2

Q30. unit vector in the direction of $u = (1, 0, 7)$

$$u = \frac{(1,0,7)}{\sqrt{1^2+0^2+49}} = \frac{(1,0,7)}{\sqrt{50}} = \left(\frac{1}{\sqrt{50}}, 0, \frac{7}{\sqrt{50}}\right)$$

Q45. which of the following is a parametrization of the line through $P=(4,9,8)$ perpendicular to the xz plane

(a) $r(t) = (4,9,8) + t(1,0,1)$

(b) $r(t) = (4,9,8) + t(0,0,1)$

(c) ✓ $r(t) = (4,9,8) + t(0,1,0)$

(d) $r(t) = (4,9,8) + t(1,1,0)$

$\because$ perpendicular to xz plane

$\therefore$ vector is $(0,1,0)$

$\therefore$ (c) is correct.

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Exercise 12.3

Q22. find the cosine of the angle between the vectors $3i+k$, $i+j+k$

Ans: $u=3i+k=(3,0,1)$

$v=i+j+k=(1,1,1)$

$||u||=\sqrt{10}$

$||v||=\sqrt{3}$

$u\cdot v=3+0+1=4$

$\therefore \cos\theta=\frac{4}{\sqrt{10}\cdot\sqrt{3}}=\frac{2\sqrt{30}}{15}$

Q42. use the properties of the dot product to evaluate the expression, assuming $u\cdot v=2$ $||u||=1$ $||v||=3$

$(u+v)\cdot(u-v)$?

Ans: $(u+v)\cdot(u-v)=u^2-v^2=1-9=-8$

Exercise 12.4

Q22. $u\times v=(1,1,0)$ $u\times w=(0,3,1)$ $v\times w=(2,-1,1)$

$(v+w)\times(3u+2v)=$?

Ans: $v\times 3u+v\times 2v+w\times 3u+w\times 2v$

$=3(-1,-1,0)+2v^2+3(0,-3,-1)+2(-2,1,-1)$

$=(-3,-3,0)+2(u\times v)\cdot(v\times w)\div(u\times w)+(-4,-7,-5)$

$=(-7,-10,-5)+2\left(\frac{2,-1,0}{0,3,1}\right)$

$=(-7,-10,-5)+(0,-\frac{2}{3},0)$

$=(-7,-\frac{32}{3},-5)$

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Q34. calculate the scalar triple product $u \cdot (v \times w)$
where $u = (1,1,0)$ $v = (3,-2,2)$ $w = (4,-1,2)$
Ans: $(1,1,0) \cdot (3,-2,2) \times (4,-1,2)$
$(3,-2,2) \times (4,-1,2)$
$= \begin{vmatrix} 3 & -2 & 2 \\ 4 & -1 & 2 \end{vmatrix}$
$= (-4+2, -6+8, -3+8)$
$= (-2, +2, 5)$
$(1,1,0) \cdot (-2,+2,5)$
$= (-2, +2, 0)$

Exercise 12.5
Q18. find an equation of the plane passing through the three points given.
$P = (5,1,1)$ $Q = (1,1,2)$, $R = (2,1,1)$
Ans: $\vec{PQ} = (-4,0,1)$ $PR = (-3,0,0)$
$\vec{PQ} \times \vec{PR} = \begin{vmatrix} -4 & 0 & 1 \\ -3 & 0 & 0 \end{vmatrix} = -3j = (0,-3,0)$
~~$0 \cdot 5 +$~~ $0 \cdot 5 + 1 \cdot (-3) + 1 \cdot 0 = -3$
$1 \cdot 0 + 1 \cdot (-3) + 2 \cdot (0) = -3$
$\therefore -3y = -3$
$3y = 3$
$y = 1$

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Q41. find the intersection of the line and plane

$z = 12$, $r(t) = t(-6, 9, 36)$

Ans: $r(t) = (-6t, 9t, 36t)$

~~$= (2t, 3t, 12t)$~~

when $z = 12$, $t = \cancel{1}3$

$\therefore (-2, 3, 12)$

Exercise 13.1

Q19. determine the radius, center and plane containing the circle.

$r(t) = (\sin t, 0, 4 + \cos t)$

Ans: $x = \sin t$, $y = 0$ $z = 4 + \cos t$

t	x	z
0	0	5
$\frac{\pi}{2}$	1	~~0~~ 4
π	0	3
$\frac{3}{2}\pi$	-1	4
2π	0	5

→ xz plane

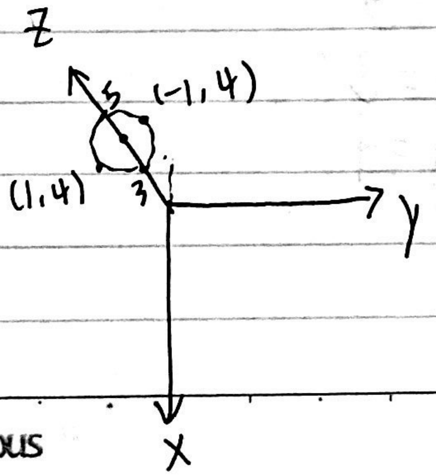
$\therefore$ radius = 1

center = (0, 0, 4)

$5 - 3 = 2$

$3 + 1 = 4$

$\therefore (0, 0, 4)$



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Q9. determine whether the space curve given by $r(t) = (t, t^3, t^2+1)$ intersects the xy plane.

Ans. $\because$ xy plane

$\therefore z = 0$

$t^2 + 1 = 0$

$t^2 = -1$

$\therefore$ DNE

the answer is not intersect.

Exercise 13.2

Q9. compute the derivative

$r(s) = (e^{3s}, e^{-s}, s^4)$

Ans: $r'(s) = (3e^{3s}, -e^{-s}, 4s^3)$

Q17. evaluate the derivative by using the appropriate Product rule $r_1(t) = (t^2, t^3, t)$ $\quad r_2(t) = (e^{3t}, e^{2t}, e^t)$

$\frac{d}{dt}(r_1(t) \cdot r_2(t))$

Ans: $\cancel{\left| \begin{matrix} t^2 & t^3 & t \\ e^{3t} & e^{2t} & e^t \end{matrix} \right|} \Rightarrow r_1(t) \cdot r_2(t)$

$= (t^2 \cdot e^{3t}, t^3 \cdot e^{2t}, te^t)$

$\frac{d}{dt}(r_1(t), r_2(t)) = (2te^{3t} + 3t^2e^{3t}, 3t^2e^{2t} + 2t^3e^{2t},$

$e^t + te^t)$

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Exercise 13.3

~~Q16~~. find the speed of the given value of t

~~$r(t) = (\cosh t, \sinh t, t) \quad t=0$~~

Exercise 13.3

Q 14. find the speed at given value of t,

$$r(t) = (e^{t-3}, 12, 3t^{-1}) \quad t=3$$

Ans: $r'(t) = v(t) = (e^{t-3}, 0, -3t^{-2})$

$$v(3) = (e^0, 0, -\tfrac{1}{3})$$

$$= (1, 0, -\tfrac{1}{3})$$

$$|v(3)| = \sqrt{1+0+\tfrac{1}{9}}$$

$$= \sqrt{\tfrac{10}{9}} = \frac{\sqrt{10}}{3}$$

Q5. compute the length of the curve over the given interval.

$$r(t) = (t, 4t^{\frac{3}{2}}, 2t^{\frac{3}{2}}) \quad 0 \le t \le 3$$

Ans: $r'(t) = (1, 6t^{\frac{1}{2}}, 3t^{\frac{1}{2}})$

$$|r'(t)| = \sqrt{1+36t+9t} = \sqrt{1+45t}$$

$$\int_0^3 |r'(t)|\,dt = -\frac{2}{135} + \frac{544\sqrt{34}}{135}$$

$$\approx 23.5$$

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Exercise 13.4

Q9. $r(t) = \langle 4t+1, 4t-3, 2t \rangle$

calculate the curvature function $k(t)$

Ans: $r'(t) = \langle 4, 4, 2 \rangle$

$r''(t) = \langle 0, 0, 0 \rangle$

$r'(t) \times r''(t) = \langle 0, 0, 0 \rangle$

$|r'(t) \times r''(t)| = 0$

$\therefore\ k(t) = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|^3} = 0$

Q 13. find the curvature at $r(t) = (\cos t, \sin t, t^2)$ $t = \frac{\pi}{2}$

Ans: $r'(t) = (-\sin t, \cos t, 2t)$

$r''(t) = (-\cos t, -\sin t, 2)$

$r'(t) \times r''(t) = (2\cos t + 2t\sin t)i - (-2\sin t + 2t\cos t)j + (\sin t^2 + \cos t^2)k$

$= (2\cos t + 2t\sin t, 2\sin t - 2t\cos t, (\sin t^2 + \cos t^2))$

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$$|r'(\tfrac{\pi}{2}) \times r''(\tfrac{\pi}{2})| = \sqrt{\pi^2 + 4 + 1}$$

$$= \sqrt{\pi^2+5}$$

$$|r'(\tfrac{\pi}{2})| = \sqrt{1+0+\pi^2} = \sqrt{1+\pi^2}$$

$$\kappa(\tfrac{\pi}{2}) = \frac{\sqrt{\pi^2+5}}{(\sqrt{1+\pi^2})^3} \approx 0.108$$

■ Exercise 13.5

Q 11. $a(t) = (t, 4)$ $v(0) = (\frac{1}{3}, -2)$

find $v(t)$ given $a(t)$ & initial v.

Ans: $v(t) = \int a(t)\,dt = \frac{1}{2}t^2 i + 4tj + C$

$$v(0) = C = \tfrac{1}{3}i - 2j$$

$$\therefore v(t) = (\tfrac{1}{2}t^2 + \tfrac{1}{3}, (4t-2)j)$$

$$= (\tfrac{3t^2+2}{6}, 4t-2)$$

Q 33. find the coefficients a_T and a_N as a function of t , $r(t) = (t, \cos t, \sin t)$

Ans: $r'(t) = v(t) = (1, -\sin t, \cos t)$

$r''(t) = a(t) = (0, -\cos t, -\sin t)$

$$\therefore a_T = \frac{a \cdot v}{|v|} = \frac{(0, -\cos t, -\sin t) \cdot (1, -\sin t, \cos t)}{\sqrt{1 + (-\sin t)^2 + (\cos t)^2}}$$

$$= \frac{0 + \sin t \cos t - \sin t \cos t}{\sqrt{1+1}} = \frac{0}{\sqrt{2}} = 0$$

$$|r'(t)| = \sqrt{1+\sin t^2+\cos t^2}$$

$$|r'(t)\times r''(t)| = \sqrt{(\sin t^2+\cos t^2)^2+\sin t^2+\cos t^2}$$

$$a_N = \frac{\sqrt{(\sin t^2+\cos t^2)^2+\sin t^2+\cos t^2}}{\sqrt{1+\sin t^2+\cos t^2}}$$

$$= \sqrt{\sin t^2+\cos t^2} = 1$$

Exercise 14.1

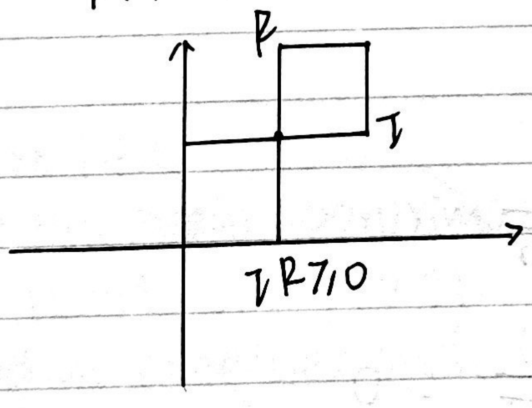
Q5. $f(x,y)=12x-5y$, sketch the domain.

Ans: because it is an linear function the domain is the xy plane.

Q11. sketch the domain of the function

$F=\sqrt{IR}$

Ans: $\because \sqrt{IR}$, $\therefore IR \geq 0$



(In this section, because the other questions need Maple to draw carefully, so I choose the first part question)

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Exercise 14.2

Q10. assume that $\lim_{(x,y)\to(2,5)} f(x,y)=3$

$\lim_{(x,y)\to(2,5)} g(x,y)=7$ find the limit

$$\lim_{(x,y)\to(2,5)} f(x,y)^2 g(x,y)$$

Ans: $= 3^2 \cdot 7$

$= 63$

Q33. evaluate the limit or determine that it DNE

$$\lim_{(x,y)\to(1,-3)} e^{x-y} \cdot \ln(x-y)$$

Ans: $= e^{1+3} \cdot \ln(1+3)$

$= e^4 \ln 4$

Q 19. $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}}$

Ans:= $\frac{(x+y)(x-y)}{\sqrt{x^2+y^2}} = 0$

(because most questions are limit, so I write 3 questions)

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Exercise 14.3

Q25. $z=\ln(x^2+y^2)$, compute the first order partial derivatives.

Ans: $\frac{\partial z}{\partial x} = 2x \cdot \frac{1}{x^2+y^2}$

$\frac{\partial z}{\partial y} = 2y \cdot \frac{1}{x^2+y^2}$

Q44. compute the given partial derivatives.

$h(x,z) = e^{xz - x^2 z^3}$ $\quad h_z(3,0)$

Ans: $\frac{\partial h}{\partial z} = (x - 3x^2 z^2) e^{xz - x^2 z^3}$

$h_z(3,0) = (3 - 3 \cdot 3^2 \cdot 0) \cdot e^{0-0}$

$= (3-0) \cdot 1$

$= 3$

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Exercise 14.4

Q16. Use the linear approximation to $f(x,y)=\sqrt{\frac{x}{y}}$ at (9,4) to estimate $\sqrt{\frac{9.1}{3.9}}$

Ans: $\frac{d}{dx} = \frac{1}{2}\cdot\left(\frac{x}{y}\right)^{-\frac{1}{2}}$ $\qquad$ $\frac{d}{dy} = -\frac{1}{2}\left(\frac{x}{y}\right)^{-\frac{1}{2}}$

$f_x(9,4) = \frac{1}{y}\cdot\frac{1}{2}\cdot\left(\frac{9}{4}\right)^{-\frac{1}{2}} = \frac{1}{3y} = \frac{1}{12}$

$f_y(9,4) = \cancel{-\frac{1}{2}(-1)x\cdot y^{-2}\cdot\left(\frac{x}{y}\right)^{-\frac{1}{2}} = -\frac{3}{8}} \quad -\frac{3}{16}$

$\therefore f'(x,y) = f'(9,4) = \cancel{\left(\frac{1}{3}, -\frac{1}{3}\right)}$

$f(9,4) = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$L(9,4) = \frac{3}{2} + \frac{1}{12}(x-9) + \cancel{\left(-\frac{1}{3}\right)}\left(-\frac{3}{16}\right)(y-4)$

$= \cancel{\frac{1}{3}x - \frac{1}{3}y - \frac{1}{6}}$

$\frac{3}{2} + \frac{1}{12}x - \frac{3}{4} - \frac{3}{16}y + \frac{3}{4}$

$= \frac{3}{2} + \frac{1}{12}x - \frac{3}{16}y$

$L(9.1, 3.9) \approx 1.527$

$\sqrt{\frac{9.1}{3.9}} \approx 1.5275$

$\therefore$ similar.

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Q 2b. $\frac{0.98^2}{2.01^3+1}$

Ans: $\frac{x^2}{y^3+z}$

$(x,y,z)=(1,2,1)$

$\frac{d}{dx} = 2x \cdot \frac{1}{y^3+z}$ $\qquad f_x = \frac{2}{9}$

$\frac{d}{dy} = -3y^2 \frac{x^2}{(y^3+z)^2}$ $\qquad f_y = -\frac{4}{27}$

$\frac{d}{dz} = -\frac{x^2}{(y^3+z)^2}$ $\qquad f_z = \frac{1}{-81}$

$f(1,2,1) = \frac{1}{8+1} = \frac{1}{9}$

$L(1,2,1) = \frac{1}{9} + \frac{2}{9}(x-1) + (-\frac{4}{27})(y-2) + (-\frac{1}{81})(z-1)$

$L(0.98, 2.01, 1) = \cancel{\frac{8}{9}}\ \frac{1}{9} + \frac{2}{9} \cdot (-0.02) + (-\frac{4}{27}) \cdot 0.01 + 0$

$= \frac{71}{675} \approx 0.105$

$\frac{0.98^2}{2.01^3+1} = 0.1053$

∴ similar.

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Exercise 14.5

Q 16. use chain Rule to calculate $\frac{d}{dt} f(r(t))$

$f(x,y) = xe^y$, $r(t) = (t^2, t^2-4t)$ $t=0$

Ans: $\frac{d}{dt} f(r(t))$

$= \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$

$= e^y \cdot 2t + xe^y \cdot (2t-4)$

$= e^0 \cdot 2(0) + 0 \cdot e^0 \cdot (0-4)$

$= 0$

Q 23. calculate the directional derivative in the direction of v. $f(x,y) = x^2y^3$ $v = i+j$ $P = (\frac{1}{6}, 3)$

Ans: $v = \cancel{(0,0)} (1,1)$

$\nabla f = (2xy^3, 3x^2y^2)$

$\|(1,1)\| = \sqrt{2}$

$u = \frac{(1,1)}{\sqrt{2}} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$\nabla f(\frac{1}{6}, 3) = (2 \cdot \frac{1}{6} \cdot 3^3, 3 \cdot (\frac{1}{6})^2 \cdot 3^2)$

$= (9, \frac{3}{4})$

$u \cdot \nabla f = (\frac{9\sqrt{2}}{2} + \frac{3\sqrt{2}}{8}) = \frac{39\sqrt{2}}{8}$

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Exercise 14.6

Q 13. use the chain Rule to evaluate partial derivative at the point specificed

$\frac{dg}{d\theta}$ at $(r,\theta)=(2\sqrt{2}, \frac{\pi}{4})$ $g(x,y)=\frac{1}{x+y^2}$

$x=r\cos\theta \quad y=r\sin\theta$

Ans: $\frac{dg}{d\theta}=\frac{dg}{dx}\cdot\frac{dx}{d\theta}+\frac{dg}{dy}\cdot\frac{dy}{d\theta}$

$=\frac{1}{-(y^2+x)^2}\cdot(-r\sin\theta)+\frac{-2y}{(y^2+x)^2}\cdot r\cos\theta$

$\frac{dg}{d\theta}(2\sqrt{2},\frac{\pi}{4})=\frac{r}{(y^2+x)^2}\cdot(\sin\theta-2y\cos\theta)$

$=-\frac{1}{6}$

Q 28. $\frac{dw}{dz}$, $x^2w+w^3+wz^2+3yz=0$

$x^2\cdot\frac{dw}{dz}+3w^2\frac{dw}{dz}+\frac{dw}{dz}\cdot z^2+2z\cdot w+3y=0$

$\frac{dw}{dz}(x^2+3w^2+z^2)=\frac{-3y-2zw}{1}$

$\frac{dw}{dz}=\frac{-3y-2zw}{x^2+3w^2+z^2}$

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Exercise 14.7

Q9. find the critical points of function and use SDT to determine whether they are local max. mini or saddle point.

$$f(x,y) = x^3 + 2xy - 2y^2 - 10x$$

$f_x = 3x^2 + 2y - 10$

$f_y = 2x + (-4y)$

$f_{xx} = 6x$

$f_{xy} = 2$

$f_{yy} = -4$

$D = 6x \cdot (-4) - 4$

$= -24x - 4$

$\approx \cancel{-43.999 < 0}$

$-24 \cdot \frac{5}{3} - 4 = -44 < 0$

$\therefore \left(\frac{5}{3}, \frac{5}{6}\right) =$ saddle point.

$3x^2 + 2y - 10 = 0$

$2x - 4y = 0$

$2(x - 2y) = 0$

① $x = 2y$

$3(2y)^2 + 2y - 10 = 0$

$3 \cdot 4y^2 + 2y - 10 = 0$

$x = 1.6\dot{6} = \frac{5}{3}$

$y = \frac{5}{6}$

② $x = -2,\ y = -1$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= -24x - 4$

$= -24 \cdot (-2) - 4$

$= 44 > 0$

$-2 \cdot 6 = -12 < 0$

$\therefore$ local maximum at $(-2, -1)$

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Q30. $f(x,y) = 2x - y$, $0 \le x \le 1$ $0 \le y \le 3$

Ans: $(0,0)$ $f = 0$

$(0,3)$ $f = -3$

$(1,0)$ $f = 2$

$(1,3)$ $f = 2 - 3 = -1$

$\therefore$ global maximum is 2

global minimum is -3.