☆ ?hand out 12.5

problem from a previous final:

Find an equation for the plane through the point $\underset{P}{(1,0,2)}$ that contains the line.

$$r(t) = \langle 1, 1, 1\rangle + t\langle 1, -1, 0\rangle$$

simplify as much as you can!

Ans:

$$r(t) = \langle 1+t, t-1, 1\rangle$$

get two point from the line

$t=0$ $\quad$ ~~2,1,1~~ $(1,1,1)Q$

$t=1$ $\quad$ $(2,0,1)R$

$PQ = (1,1,1) - (1,0,2) = \langle 0,+1,-1\rangle$

$PR = (2,0,1) - (1,0,2) = \langle 1,0,-1\rangle$

$PQ \times PR = \langle 0,+1,-1\rangle \times \langle 1,0,-1\rangle$

$$\begin{array}{ccc} i & j & k \\ 0 & +1 & -1 \\ 1 & 0 & -1 \end{array} \quad \begin{aligned} &= i(-1-0) - j(0+1) + k(0-1) \\ &= -i - j - k \\ &= \langle -1,-1,-1\rangle \end{aligned}$$

$-1\times(x-1) - 1\times(y-0) - 1\times(z-2) = 0$

$-x+1-y-z+2=0$

$$\boxed{x+y+z=3}$$

my own problem setting:

Find an equation for the plane through the point $(5,5,5)$ (P) that contains the line

$$r(t) = \langle 2,2,2\rangle + t\langle 1,-1,0\rangle.$$

simplify as much as you can.

Ans: get two point:

$t=0$ $(2,2,2)$ (Q)

$t=1$ $(3,1,2)$ (R)

$PQ = (2,2,2)-(5,5,5) = \langle -3,-3,-3\rangle$

$PR = (3,1,2)-(5,5,5) = \langle -2,-4,-3\rangle$

$PQ \times PR = \langle -3,-3,-3\rangle \times \langle -2,-4,-3\rangle$

$$\begin{matrix} i & j & k \\ -3 & -3 & -3 \\ -2 & -4 & -3 \end{matrix}$$

$= i(9-12) - j(9-6) + k(12-6)$

$= -3i - 3j + 6k$

$= \langle -3,-3,6\rangle$

$-3(x-5) - 3(y-5) + 6(z-5) = 0$

$-3x + 15 - 3y + 15 + 6z - 30 = 0$

$$\boxed{x + y - 2z = 0}$$

handout 13.4

problem from a previous final
Find the velocity and position vectors of a particle whose acceleration is $a(t)=i+j$, and time $t=0$, the velocity is $i-j$ and position is k.

Ans: $\int a(t) = v(t)$

$\int i+j = ti+tj+C$

plug $t=0$

$C = i-j$

$v(t) = ti+tj+i-j$

$= (t+1)i+(t-1)j$

$= \langle t+1, t-1, 0 \rangle$

$\int v(t) = p(t)$

$\int (t+1)i + (t-1)j = (\frac{1}{2}t^2+t)i + (\frac{1}{2}t^2-t)j + C$

plug $t=0$

$C = k$

$p(t) = (\frac{1}{2}t^2+t)i + (\frac{1}{2}t^2-t)j + k$

own similar problem setting:

Find the velocity and position vectors of a particle whose acceleration is $a(t) = i+j+k$, and time $t=0$, the velocity $i-j+k$ and position is $2i+2j+2k$.

$$\int a(t) = v(t)$$

$$\int i+j+k = ti+tj+tk+c$$

plug $t=0$

$$c = i-j+k$$

$$\boxed{v(t) = (t+1)i+(t-1)j+(t+1)k}$$

$$\int v(t) = p(t)$$

$$\int (t+1)i+(t-1)j+(t+1)k = (\tfrac{1}{2}t^2+t)i+(\tfrac{1}{2}t^2-t)j+(\tfrac{1}{2}t^2+t)k+c$$

plug $t=0$

$$c = 2i+2j+2k$$

$$\boxed{p(t) = (\tfrac{1}{2}t^2+t+2)i+(\tfrac{1}{2}t^2-t+2)j+(\tfrac{1}{2}t^2+t+2)k}$$

handout 13.3

A problem from a previous final.

suppose that the position of a certain particle is given by

$$r(t) = \langle e^t \cos t,\ e^t \sin t,\ e^t \rangle, \quad 0 \le t \le \pi$$

(a) Find the velocity of the particle as a function of the time t.

(b) Find the length of the arc traversed by the moving particle for $0 \le t \le \pi$.

(a) $v(t) = \langle e^t\cos t - e^t\sin t,\ e^t\sin t + e^t\cos t,\ e^t \rangle$

(b) $|v(t)| = \sqrt{(e^t(\cos t - \sin t))^2 + (e^t(\sin t + \cos t))^2 + (e^t)^2}$

Length of the arc $= \int_0^{\pi} |v(t)|\, dt$

$= \sqrt{(e^{\pi}\cdot(-1-0))^2 + (e^{\pi}(-1))^2 + (e^{\pi})^2} - \sqrt{(1\cdot(1-0))^2 + (1\cdot 1)^2 + (1)^2}$

$= \sqrt{e^{2\pi} + e^{2\pi} + e^{2\pi}} - \sqrt{1+1+1}$

$= \sqrt{3e^{2\pi}} - \sqrt{3}$

$= \sqrt{3}\, e^{\pi} - \sqrt{3}$

$= \sqrt{3}(e^{\pi} - 1)$

~~四川~~ My problem setting:

suppose that the velocity of a certain particle is given by

$$v(t) = \langle e^t, e^t, e^t \rangle \quad 0 \le t \le \pi.$$

(a) Find the ~~accerlate~~ ~~accert~~ accelerate of the particle as a function of the time t.

(b) Find the length of the arc traversed by the moving particle for ~~0≤t≤π~~ $0 \le t \le \pi$.

(a) $a(t) = v'(t) = \langle e^t, e^t, e^t \rangle$

(b)

Length of the arc $= \int_0^{\pi} \|v(t)\| dt$

$\|v(t)\| = \sqrt{e^{2t}+e^{2t}+e^{2t}} = \sqrt{3}e^t$.

$$\int_0^{\pi} \sqrt{3}e^t dt = \sqrt{3}e^t \Big|_0^{\pi} = \sqrt{3}(e^{\pi}-e^0)$$
$$= \sqrt{3}(e^{\pi}-1)$$

Ans: $\sqrt{3}(e^{\pi}-1)$

handout 13.4

Problem from a previous final Exam.

Find the curvature of the curve.

$$r(t) = \langle t, t^2, \tfrac{2}{3}t^3 \rangle$$

at the point $(1, 1, \tfrac{2}{3})$

$t=1 \quad t^2=1 \quad \tfrac{2}{3}t^3 = \tfrac{2}{3} \longrightarrow t=1.$

$r'(t) = \langle 1, 2t, 2t^2 \rangle$

$r''(t) = \langle 0, 2, 4t \rangle$

$r'(t) \times r''(t) = (8t^2 - 4t^2)i - (4t - 0)j + (2-0)k.$

$= (4t^2)i - (4t)j + 2k.$

$$\begin{matrix} i & j & k \\ 1 & 2t & 2t^2 \\ 0 & 2 & 4t \end{matrix}$$

$|r'(t) \times r''(t)| = \sqrt{16t^4 + 16t^2 + 4} = \sqrt{(4t^2+2)^2} = 4t^2+2$

$= 4+2$

$= 6.$

$|r'(t)|^3 = \left(\sqrt{1 + 4t^2 + 4t^4}\right)^3 = \left(\sqrt{(2t^2+1)^2}\right)^3 = (2t^2+1)^3$

$= (2+1)^3$

$= 3^3.$

$= 27.$

$k(t) = \dfrac{\cancel{6}\,2}{\cancel{27}\,9} = \dfrac{2}{9}.$

Ans: $\dfrac{2}{9}.$

my problem setting:

Find the curvature of the curve

$$r(t) = \langle 2t, 2t^2, \tfrac{4}{3}t^3 \rangle$$

at the point $(2, 2, \frac{4}{3})$

$2t=2 \quad 2t^2=2 \quad \frac{4}{3}t^3 = \frac{4}{3} \longrightarrow t=1.$

$r'(t) = \langle 2, 4t, 4t^2 \rangle$

$r''(t) = \langle 0, 4, 8t \rangle$

$r'(t) \times r''(t) = i(32t^2 - 16t^2) - j(16t - 0) + k(8-0)$

$\begin{matrix} i & j & k \\ 2 & 4t & 4t^2 \\ 0 & 4 & 8t \end{matrix}$

$= (16t^2)i - (16t)j + 8k.$

$= 16i - 16j + 8k.$

$|r'(t) + r''(t)| = \sqrt{256 + 256 + 64} = 24.$

$|r'(t)| = |\langle 2, 4, 4 \rangle| = \sqrt{4+16+16} = \sqrt{36} = 6.$

$$k = \frac{|r'(t) + r''(t)|}{|r'(t)|^3} = \frac{24}{216} = \frac{1}{9}.$$

Ans: $\frac{1}{9}$.

handout 14.2

problem from a previous final

Compute the limit:

$$\lim_{(x,y,z)\to(1,1,1)} e^{-xy}\sin(\pi z/2).$$

or prove that it does not exist.

Ans:

$$e^{-1}\sin\left(\frac{\pi}{2}\right)=\frac{1}{e}\times 1=\frac{1}{e}.$$

It ~~is~~ exists and equals ~~$\frac{1}{e}$~~ $\frac{1}{e}$.

My problem setting:

Compute the limit:

$$\lim_{(x,y,z)\to(2,2,2)} e^{-xy}\sin(\pi z/2)$$

or prove that it does not exist.

Ans:

$$e^{-4}\sin\left(\frac{2}{2}\pi\right)$$

$$=e^{-4}\cdot 0$$

$$=0$$

It exists and equals 0.

☆7 handout 14.4

problem from a previous final:

Find an equation of the tangent plane to the surface

$$z=e^{2x-3y}$$

at the point (3,2,1). simplify as much as you can!

$$f_x = 2e^{2x-3y} \qquad f_y = -3e^{2x-3y}$$

$$= 2e^{6-6} \qquad = -3e^{0}$$

$$= 2e^{0} \qquad = -3$$

$$= 2$$

$$z-1 = 2(x-3)+(-3)(y-2)$$

$$z = 2x-6-3y+6+1$$

$$\boxed{z = 2x-3y+1.}$$

my own problem setting:

Find an equation of the tangent plane to the surface

$$z = 5x^2+3y^2$$

at the point (3,2,1) simplify as much as you can!

$$f_x = 10x \qquad f_y = 6y \qquad z-1 = 30(x-3)+12(y-2)$$

$$= 30 \qquad = 12 \qquad z = 30x-90+12y-24+1$$

$$\boxed{z = 30x+12y-113}$$

7 handout 14.5

A problem from a previous final

Let

$$f(x,y,z) = -x^2+y^2+z^2-1.$$

(a) compute ∇f

(b) Find a normal to the level surface $f(x,y,z)=0$ at the point $(1,1,1)$, and give an equation for the tangent plane to that surface at that point.

(c) compute the directional derivative of $f(x,y,z)$ at the point $(1,1,1)$ in the direction $\langle 1,2,2\rangle$.

(a) $\nabla f = \langle -2x, 2y, 2z\rangle$

7(b) $f_x = -2x$ $\quad f_y = 2y$ $\quad f_z = 2z$
$= -2 \quad = 2 \quad = 2$

$\langle -2, 2, 2\rangle$

$f = -2(x-1)+2(y-1)+2(z-1)$
$= -x+1+y-1+z-1$
$= -x+y+z-1$
$\boxed{z = x-y+1}$

(c) $\nabla f = \langle -2, 2, 2\rangle$

unit $= \dfrac{\langle 1,2,2\rangle}{\sqrt{4+4+1}} = \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\rangle$

$\langle -2,2,2\rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\rangle$
$= -\frac{2}{3}+\frac{4}{3}+\frac{4}{3}$
$= \frac{8-2}{3} = \frac{6}{3} = 2$

$\boxed{\text{Ans } 2}$

my own problem setting:

Let

$$f(x,y,z) = x^3 + y^3 + z^3 - 1$$

(a) Compute ∇f

(b) Find a normal to the level surface $f(x,y,z) = 0$ at the point $(1,-1,1)$, and given an equation for the tangent plane to that surface at that point.

(c) compute the directional derivative of $f(x,y,z)$ at the point $(1,1,1)$ in the direction $\langle 1,2,2 \rangle$

(a) $\nabla f = \langle 3x^2, 3y^2, 3z^2 \rangle$

(b) $f_x = 3x^2 = 3$ $\quad f_y = 3y^2 = 3$ $\quad f_z = 3z^2 = 3$

$\langle 3, 3, 3 \rangle$

$f = 3(x-1) + 3(y+1) + 3(z-1)$

$= x - 1 + y + 1 + z - 1$

$= x + y + z - 1$

$-z = x + y - 1$

$\boxed{z = -x - y + 1}$

(c)

$\nabla f = \langle 3, 3, 3 \rangle$

$u = \dfrac{\langle 1,2,2 \rangle}{3}$

$= \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$\nabla f \cdot u = 1 + 2 + 2$

$= 5$

$\boxed{\text{directional derivative} = 5}$

handout 14.6

A problem from a previous final

Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as functions of r and s, if

$$f(x,y) = x^3 + 2xy + y^3$$

and the variables are related by $x = r - s$ and $y = r + s$.
You do not need to simplify!

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$\frac{\partial f}{\partial x} = 3x^2 + 2y \qquad \frac{\partial f}{\partial y} = 2x + 3y^2$$

$$\frac{\partial x}{\partial r} = 1 \qquad \frac{\partial y}{\partial r} = 1$$

$$\frac{\partial x}{\partial s} = -1 \qquad \frac{\partial y}{\partial s} = 1$$

$$\frac{\partial f}{\partial r} = (3x^2 + 2y) \cdot 1 + (2x + 3y^2) \cdot 1 = 3x^2 + 3y^2 + 2y + 2x$$
$$= 3(r-s)^2 + 3(r+s)^2 + 2(r+s) + 2(r-s)$$

$$\frac{\partial f}{\partial s} = (3x^2 + 2y) \cdot (-1) + (2x + 3y^2) \cdot 1 = -3x^2 - 2y + 2x + 3y^2$$
$$= -3(r-s)^2 - 2(r+s) + 2(r-s) + 3(r+s)^2$$

Another problem from a previous final

Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if

① $$\sin(x+2y+3z)=5xyz+1$$

$\frac{dz}{dx}$: $\cos(x+2y+3z)\cdot(1+3z') = 5yz+5xyz'$

$$\cos(x+2y+3z)+3z'\cdot\cos(x+2y+3z) = 5yz+5xyz'$$

$$[3\cos(x+2y+3z)-5xy]\,z' = 5yz-\cos(x+2y+3z)$$

$$\frac{dz}{dx} = z' = \frac{(5yz-\cos(x+2y+3z))}{(3\cos(x+2y+3z)-5xy)}$$

$\frac{dz}{dy}$: $\cos(x+2y+3z)\cdot(2+3z') = 5xz+5xyz'$

$$2\cos(x+2y+3z)+3z'\cdot\cos(x+2y+3z) = 5xz+5xyz'$$

$$z'\cdot(3\cos(x+2y+3z)-5xy) = [5xz-2\cos(x+2y+3z)]$$

$$\frac{dz}{dy} = z' = \frac{[5xz-2\cos(x+2y+3z)]}{[3\cos(x+2y+3z)-5xy]}$$

14.6 handout

my own problem setting

① Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as function of r and s, if

$$f(x, y) = x^2 + y^2$$

and the variable are related by $x = rs$ and $y = 2r - s$.

You do not need to simplify!

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}.$$

$\frac{\partial f}{\partial x} = 2x$ $\qquad$ $\frac{\partial f}{\partial y} = 2y$

$\frac{\partial x}{\partial r} = s$ $\qquad$ $\frac{\partial y}{\partial r} = 2$

$\frac{\partial x}{\partial s} = r$ $\qquad$ $\frac{\partial y}{\partial s} = -1$

$$\frac{\partial f}{\partial r} = 2x \cdot s + 2y \cdot 2$$
$$= 2s \cdot (rs) + 4(2r - s)$$
$$= 2rs^2 + 4(2r - s)$$

$$\frac{\partial f}{\partial s} = 2x \cdot r + 2y \cdot -1$$
$$= 2 \cdot rs \cdot r - 2 \cdot (2r - s)$$
$$= 2r^2 s - 2(2r - s)$$

(2) Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if

$$x^2 + y^2 + z^2 = 5xyz + 1$$

$\frac{dz}{dx}$: $2x + 2zz' = 5yz + 5xyz'$

$(2 - 5xy)z' = 5yz - 2x$

$$\frac{dz}{dx} = z' = \frac{5yz - 2x}{(2 - 5xy)}$$

$\frac{dz}{dy}$: $2y + 2z' = 5xz + 5xyz'$

$(2 - 5xy)z' = 5xz - 2y$

$$\frac{dz}{dy} = z' = \frac{5xz - 2y}{2 - 5xy}$$

handout #1.7

problem from a previous final

Find the local maximum and minimum points, the local maximum and minimum values, and saddle points of the function.

$$f(x,y) = 4x^2 + y^2 + 2x^2y - 1.$$

Ans: $f_x = 8x + 4xy$ $\quad f_y = 2y + 2x^2$

$f_{xx} = 8 + 4y$ $\quad f_{yy} = 2$

$f_{xy} = 4x$

$8x + 4xy = 0$ $\quad 2y + 2x^2 = 0$

$x = 0, \pm\sqrt{2}$ $\quad y = 0, -2$

$D = [(8-8)\cdot 2 - (4\cdot\sqrt{2})^2] < 0$

$D = [(8-8)\cdot 2 - (4\cdot -\sqrt{2})^2] < 0$

critical point $(0,0)$

$f(0,0) = -1$ $\quad f(\sqrt{2},-2)$; $f(-\sqrt{2},-2)$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 8 \cdot 2 - 0$

$= 16 > 0$

$D > 0$ $\quad f_{xx} = 8 > 0$

$(0,0)$ local min

Ans: $(0,0)$ is local min point

value: -1.

point $(\sqrt{2},-2)$, point $(-\sqrt{2},-2)$ are saddle point.

Another problem from a previous final
Find the local maximum and minimum point(s), the local maximum and minimum values, and saddle point(s) of the function.

$$f(x,y) = 6y^2 - 2y^3 + 3x^2 + 6xy$$

$f_x = 6x + 6y$ $f_y = 12y - 6y^2 + 6x$

$f_{xx} = 6$ $f_{yy} = 12 - 12y$

$f_{xy} = 6$ $D(0,0) = 72 - 36 = 36 > 0$

$f_{xx} > 0$

local min

value: 0

$\begin{cases} 6x + 6y = 0 \\ 12y - 6y^2 + 6x = 0 \end{cases}$ $D(-1,+1) = 6 \cdot 0 - 36$

$= -36 < 0$

$y_1 = 0$ $y_2 = 1$

$x_1 = 0$ $x_2 = -1$

$(0,0)$ $(-1,+1)$ $(-1,+1)$ saddle point.

Local max: none

Ans: $(0,0)$ local min

value: 0

$(-1,+1)$ saddle point.

handout 14.7

my own problem setting:

Find the local maximum and minimum points, the local maximum and minimum values, and saddle points of the function.

$$f(x,y) = 7x^2 + 4y + 2xy^2$$

$f_x = 14x + 2y^2$ $\qquad$ $f_y = 4 + 4xy$

$f_{xx} = 14$ $\qquad$ $f_{yy} = 4x$

$f_{xy} = 4y$

$$\begin{cases} 14x + 2y^2 = 0 \\ 4 + 4xy = 0 \end{cases}$$

$y_1 = 0 \quad y_2 = 7$

$x_1 = 0 \quad x_2 = -\frac{1}{7}$

$(0,0) \quad (-\frac{1}{7}, 7)$

$D = 14 \cdot 0 - (0)^2$
$= 0$

~~undec~~

$\boxed{(0,0) \text{ we don't know}}$

$D = 14 \cdot 4 \cdot -\frac{1}{7} - (4 \times 7)^2$
$= -\frac{56}{7} - 28 < 0$

$\boxed{(-\frac{1}{7}, 7) \text{ is saddle point}}$

Because the previous is same type, we just have one own setting problem.

12.1

14. calculate

$\langle -4,6\rangle - \langle 3,2\rangle = \langle -7, 8\rangle$

42. Unit vector e_w where $w=\langle 24,7\rangle$

$e_w = \frac{\langle 24,7\rangle}{25} = \langle \frac{24}{25}, \frac{7}{25}\rangle$

12.2

20. calculate the linear combination.

$6(4j+2k) - 3(2i+7k)$

$= 24j + 12k - 6i - 21k$

$= -6i + 24j - 9k$

26. determine whether or not the two vectors are parallel.

$u = \langle 4, 2, -6\rangle \qquad v = \langle 2, 1, 3\rangle$

$\frac{4}{2} = 2$

$\frac{2}{1} = 2$

$\frac{-6}{3} = -2 \neq 2$

not parallel.

12.3

22. find the cosine of the angle between the vectors

$3i+k$ $\quad$ $i+j+k$

$\langle 3,0,1\rangle$ $\quad$ $\langle 1,1,1\rangle$

$\cos\theta = \frac{\langle 3,0,1\rangle \cdot \langle 1,1,1\rangle}{\sqrt{9+1}\cdot\sqrt{3}} = \frac{3+0+1}{\sqrt{30}} = \frac{4}{\sqrt{30}} = \frac{\frac{2}{\cancel{4}\sqrt{30}}}{\frac{\cancel{30}}{15}} = \frac{2}{15}\sqrt{30}$

~~33~~ 30. find a vector that is orthogonal to $\langle -1,2,2\rangle$

$\langle 0,-1,1\rangle$ is orthogonal to $\langle -1,2,2\rangle$

$\langle 0,-1,+1\rangle \cdot \langle -1,2,2\rangle$

$= 0+(-2)+2 = 0$.

$\therefore \langle 0,-1,1\rangle$ is orthogonal to $\langle -1,2,2\rangle$.

12.4

2. calculate:

$$\begin{vmatrix} \frac{2}{3} & \frac{1}{6} \\ -5 & 2 \end{vmatrix} = \frac{2}{3}\cdot 2 - \frac{1}{6}\cdot(-5)$$

$$= \frac{4}{3} + \frac{5}{6}$$

$$= \frac{8+5}{6} = \frac{13}{6}$$

38. calculate the volume of the parallelepiped spanned by

$u=\langle 2,2,1\rangle$ $\quad$ $v=\langle 1,0,3\rangle$ $\quad$ $w=\langle 0,-4,0\rangle$

$u\times v = 6i-5j+(-2)k = \langle 6,-5,-2\rangle$ $\quad$ $\langle +6,-5,-2\rangle \cdot \langle 0,-4,0\rangle = 0+20+0 = 20$ $\quad$ volume: 20.

12.5

18. Find an equation of the plane passing through the three points given.

$P=(5,1,1) \quad Q=(1,1,2) \quad R=(2,1,1)$

$PQ=(1,1,2)-(5,1,1)=(-4,0,1)$

$PR=(2,1,1)-(5,1,1)=(3,0,0)$

$PQ \times PR=(-4,0,1)\times(3,0,0)$

$=3j=\langle 0,3,0\rangle$

$0\cdot(x-5)+3\cdot(y-1)+0\cdot(z-1)=0$

$3y-3=0$

$3y=3$

$\boxed{y=1}$

14. find a vector normal to the plane with the given equation.

$x-z=0$

$\boxed{\langle 1,0,-1\rangle}$

713.1

The function r(t) traces a circle. Determine the radius, center, and plane containing the circle.

?18. $r(t) = 7i + (12\cos t)j + (12\sin t)k$.

$= \langle 7, 0, 0 \rangle + 12\langle 0, \cos t, \sin t \rangle$

center: $\langle 7, 0, 0 \rangle$
radius: 12
plane: yz-plane

I dont understand this one.

?19. $r(t) = (\sin t, 0, 4 + \cos t)$
$= \langle 0, 0, 4 \rangle + \langle \sin t, 0, \cos t \rangle$

radius: 1
center: (0,0,4)
plane: xz-plane.

I dont understand this one.

13.2

2. $\lim_{t\to\pi} \sin 2t\, i + \cos t\, j + \tan 4t\, k$

$= \sin 2\pi\, i + \cos\pi\, j + \tan 4\pi\, k$

$= 0\, i + (-1) j + \tan 4\pi\, k$

$= \langle 0, -1, \tan 4\pi \rangle$

8. $r(t) = \langle 7-t, 4\sqrt{t}, 8 \rangle$

$r'(t) = \langle -1, \frac{2}{\sqrt{t}}, 0 \rangle$

13.3

4. $r(t) = \langle \cos t, \sin t, t^{\frac{3}{2}} \rangle \quad 0 \le t \le 2\pi.$

$r'(t) = \langle -\sin t, \cos t, \frac{3}{2} t^{\frac{1}{2}} \rangle$

$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t + \frac{9}{4} t}$

$= \sqrt{1 + \frac{9}{4} t}$

$\int_0^{2\pi} \sqrt{1+\frac{9}{4}t}\, dt$

$= \cancel{\sqrt{}} \quad \frac{8}{27}(1+\frac{9}{4}t)^{\frac{3}{2}} \Big|_0^{2\pi}$

$= \frac{8}{27}(1+\frac{9}{\cancel{4}2}\pi)^{\frac{3}{2}} - \frac{8}{27} \times 1$

$= \frac{\cancel{8}4}{\cancel{27}} \times \frac{\cancel{27}1}{\cancel{2}\sqrt{2}} \pi^{\frac{3}{2}}$

$= 2\sqrt{2}\pi^{\frac{3}{2}}$

≈ 15.75

12. $r(t) = \langle t, t^2, t^3 \rangle \; t=1$ find speed.

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$|r'(t)|_{t=1} = \sqrt{1+4t^2+9t^4}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$= \sqrt{14}.$

speed: $\sqrt{14}$.

13.4

3、$r(t)=\langle 3+4t, 3-5t, 9t\rangle$ find $r'(t)$ and $T(t)$, evaluate $T(1)$.

$r'(t)=\langle 4, -5, 9\rangle$

$$T(t) = \frac{\langle 4,-5,9\rangle}{\sqrt{16+25+81}}$$

$$T(1) = \frac{\langle 4,-5,9\rangle}{\sqrt{122}}$$

10、Calculate the curvature function $k(t)$

$r(t) = \langle t^{-1}, 1, t\rangle$

$r'(t) = \langle -t^{-2}, 0, 1\rangle$

$r''(t) = \langle 2t^{-3}, 0, 0\rangle$

$$r'(t)\times r''(t) = i(0-0) - j(0-2t^{-3}) + k(0-0)$$

$$\begin{vmatrix} i & j & k \\ -t^{-2} & 0 & 1 \\ 2t^{-3} & 0 & 0 \end{vmatrix} = 2t^{-3} j$$

$$|r'(t)\times r''(t)| = \sqrt{(2t^{-3})^2} = 2t^{-3}$$

$$|r'(t)| = \sqrt{(-t^{-2})^2+1} = \sqrt{t^{-4}+1}$$

$$k(t) = \frac{2t^{-3}}{(\sqrt{t^{-4}+1})^3}$$

4. calculate the velocity and acceleration vectors and the speed at the time indicated.

$r(t) = e^t j - \cos(2t)k, t=0$

$v(t) = r'(t) = e^t j + 2\sin(2t)k$

$t=0$

$v(0) = e^0 j + 2\sin(0)k = j$

speed $= \sqrt{e^{2t} + (2\sin 2t)^2} = \sqrt{1+0} = 1.$

$a(t) = v'(t) = r''(t) = e^t j + 4\cos 2t\, k.$

$a(0) = e^0 j + 4\cos \cancel{2t}\, k = j + 4k.$

16. Find $r(t)$ and $v(t)$ given $a(t)$ and the initial velocity and position.

$a(t) = \langle e^t, 2t, t+1 \rangle \quad v(0) = \langle 1, 0, 1 \rangle \quad r(0) = \langle 2, 1, 1 \rangle$

$v(t) = \langle e^t, t^2, \frac{1}{2}t^2 + t \rangle$

$v(0) = \langle e^0, 0^2, 0+0 \rangle$

$= \langle 1, 0, 0 \rangle$

$C = k.$

$v(t) = \langle e^t, t^2, \frac{1}{2}t^2 + t + 1 \rangle$

$r(t) = \langle e^t, \frac{1}{3}t^3, \frac{1}{6}t^3 + \frac{1}{2}t^2 + t \rangle \qquad r(t) = \langle e^t + 1, \frac{1}{3}t^3 + 1, \frac{1}{6}t^3 + \frac{1}{2}t^2 + t + 1 \rangle$

$r(0) = \langle e^0, \frac{1}{3}\cdot 0, 0 \rangle$

$= \langle 1, 0, 0 \rangle$

$C = i + j + k.$

14.1. 6. sketch the domain of the function

2. $g(x,y)=\frac{y}{x^2+y^2}$, $(1,3)$, $(3,-2)$

$g(1,3)=\frac{3}{1+9}=\frac{3}{10}$

$g(3,-2)=\frac{-2}{9+4}=\frac{-2}{13}$

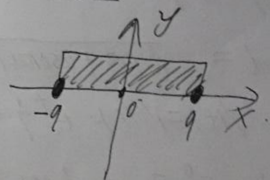
6. $f(x,y)=\sqrt{81-x^2}$

$81-x^2\geq 0$

$-x^2\geq -81$

$x^2\leq 81$

$\boxed{-9\leq x\leq 9}$



14.2

evaluate the limit or determine that it does not exist.

29. $\lim_{(x,y)\to(4,2)}\frac{y-2}{\sqrt{x^2-4}}=\frac{2-2}{\sqrt{16-4}}=0$

30. $\lim_{(x,y)\to(0,0)}\frac{x^2+y^2}{1+y^2}=\frac{0}{1}=0$

14.3

compute the first-order partial derivatives

14. $z=x^4y^3$

$\frac{dz}{dx}=4x^3y^3$

$\frac{dz}{dy}=3x^4y^2$

$\cancel{z=\frac{x}{x-y}}$

$\cancel{\frac{dz}{dx}}$

29. $z=e^{xy}$

$\frac{dz}{dx}=ye^{xy}$

$\frac{dz}{dy}=xe^{xy}$

14.4

4, find an equation of the tangent plane at the given point.

$f(x,y) = \dfrac{x}{\sqrt{y}} \quad (4,4)$

$\dfrac{dz}{dx} = \dfrac{1}{\sqrt{y}} = \dfrac{1}{2}$

$\dfrac{dz}{dy} = -\dfrac{1}{2} x y^{-\frac{3}{2}} = -\dfrac{1}{4}$

$z = \frac{1}{2}(x-4) + (-\frac{1}{4})(y-4)$

$z = \frac{1}{2}(x-4) - \frac{1}{4}(y-4).$

18, Let $f(x,y) = \dfrac{x^2}{(y^2+1)}$, Use the linear approximation at an appropriate point (a,b) to estimate $(4.01, 0.98)$

$(a,b) = (4,1)$

$\dfrac{x^2}{(y^2+1)}$ is continuous

$\dfrac{dz}{dx} = \dfrac{2x}{y^2+1} = \dfrac{\cancel{8}\,4}{\cancel{2}\,1} = 4$

$\dfrac{dz}{dy} = \cancel{\dfrac{x^2}{y^2+1}} \quad -2x^2y\cdot(y^2+1)^{-2} \quad \cancel{2x^{-2}y^{-3}(y^2+1)^{-2}}$

$= -2\times 16\times 1\cdot(1+1)^{-2}$

$= -32\times\dfrac{1}{4}$

$= -8$

Lineration $= 8 + 4(x-4) - 8\times(y-1)$

$f(4.01, 0.98) = 8.2$

14.5

use the chain rule to calculate $\frac{d}{dt} f(r(t))$

10, $f(x,y) = 3x - 7y$, $r(t) = \langle t^2, t^3 \rangle$, $t = 2$.

~~$r(2) = \langle 4, 8 \rangle$~~

~~$f(4,8) =$~~ ~~$\frac{dx}{d}$~~

$\frac{df}{dx} = 3$ $\frac{df}{dy} = -7$

$\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = 3t^2$.

$= 4$ $= 12$.

$$\frac{df}{dt} = 3 \cdot 4 + (-7) \cdot 12$$
$$= 12 - 84$$
$$= -72.$$

28, calculate the directional derivative in the direction of v at the given point. Remember to normalize the direction vector.

$g(x,y,z) = z^2 - xy^2$, $v = (-1, 2, 2)$ $P = (2, 1, 3)$

$\nabla f = \langle -y^2, -2xy, 2z \rangle = \langle -1, -4, 6 \rangle$

unit $v = \frac{(-1,2,2)}{3} = \langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$\nabla f \cdot u = \langle -1, -4, 6 \rangle \cdot \langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$= \frac{1}{3} - \frac{8}{3} + 4$

$= \frac{5}{3}$

14.6.

use the Chain Rule to calculate the partial derivatives. Express the answer in terms of the independent variables

4, $\frac{df}{dr}$ $\frac{df}{dt}$; $f(x,y,z) = xy + z^2$, $x = r + s - 2t$

$y = 3rt$, $z = s^2$

$\frac{df}{dx} = y$ $\frac{df}{dy} = x$ $\frac{df}{dz} = 2z$.

$\frac{dx}{dr} = 1$ $\frac{dy}{dr} = 3t$ $\frac{dz}{dr} = 0$.

$\frac{dx}{dt} = -2$ $\frac{dy}{dt} = 3r$ $\frac{dz}{dr} = 0$

$$\frac{df}{dr} = y \cdot 1 + x \cdot 3t + 0$$
$$= 3rt + 3rt + 3st - 6t^2$$
$$= 6rt + 3st - 6t^2.$$

$$\frac{df}{dt} = y \cdot (-2) + x \cdot 3r + 0$$
$$= -6rt + 3r^2 + 3rs - 6rt$$
$$= -12rt + 3r^2 + 3rs.$$

16. Use the chain Rule to evaluate the partial derivative at the point specified.

16. $\frac{\partial h}{\partial q}$ at $(q,r)=(3,2)$ where $h(u,v)=ue^v$, $u=q^3$, $v=qr^2$

$h(u,v)=ue^v=q^3e^{qr^2}$

$\frac{\partial h}{\partial q}=3q^2e^{qr^2}+q^3e^{qr^2}\cdot r^2$

plug $(q,r)=(3,2)$

$=3\cdot3^2\cdot e^{3\cdot4}+\cancel{3}\,3^3e^{3\cdot4}\cdot4$

$=27e^{12}+108e^{12}$

$=135e^{12}$

14.7

find the critical points of the function, Then use the second Derivative Test to determine whether they are local minimum, local maximum, or saddle points (or state that the test fails)

8. $f(x,y)=x^3-xy+y^3$ $(\frac{1}{3},\frac{1}{3})$, $(0,0)$

$\frac{\partial f}{\partial x}=3x^2-y$

$\frac{\partial f}{\partial y}=-x+3y^2$

$\begin{cases}3x^2-y=0\\-x+3y^2=0\end{cases}$

$x_1=\frac{1}{3}$ $x_2=0$

$y_1=\frac{1}{3}$ $y_2=0$

$f\_xx=6x$ $f\_xy=-1$

$f\_yy=6y$

$D=\cancel{6}^{2}\cdot\frac{1}{\cancel{3}}\cdot\cancel{6}^{2}\cdot\frac{1}{\cancel{3}}-(-1)^2$

$=4-1$

$=3>0$

$f\_xx=\cancel{6}^{2}\frac{1}{\cancel{3}}=2>0$

$(\frac{1}{3},\frac{1}{3})$ local min

$(0,0)$ saddle point

$D=0-(-1)^2$

$=-1<0$

14.7

13. $f(x,y)=x^4+y^4-4xy$

$\frac{df}{dx}=4x^3-4y$

$\frac{df}{dy}=4y^3-4x$

$4x^3-4y=0$

$4y^3-4x=0$

$x_1=0 \quad x_2=1 \quad x_3=-1$

$y_1=0 \quad y_2=1 \quad y_3=-1$

$(0,0) \quad (1,1) \quad (-1,-1)$

$f\_xx=12x^2$

$f\_yy=12y^2$

$f\_xy=-4$

$D=0-16$

$=-16<0$

$(0,0)$ is saddle point

$D=144-16$

$=128>0$

$f\_xx=12>0$

$(1,1)$ is local min

$D=144-16$

$=128>0$

$f\_xx=12>0$

$(-1,-1)$ is local min