

Yash Khangura "Quiz" for Lecture 9 Section 24

1.) Find $\frac{df}{dr}$ and $\frac{df}{ds}$ as functions of r and s , if

$f(x, y) = x^2 + 2xy^2 + 2y^3$ and the variables are related by $x = r + 2s$ and $y = 3r + 2s$. You do not need to simplify!

$$\frac{df}{dr} = \frac{df}{dx} \frac{dx}{dr} + \frac{df}{dy} \frac{dy}{dr}, \quad \frac{df}{ds} = \frac{df}{dx} \frac{dx}{ds} + \frac{df}{dy} \frac{dy}{ds}$$

$$\frac{df}{dx} = 2x + 2y^2 \quad \frac{dx}{dr} = 1 \quad \frac{dx}{ds} = 2$$

$$\frac{df}{dy} = 4xy + 6y^2 \quad \frac{dy}{dr} = 3 \quad \frac{dy}{ds} = 2$$

$$\frac{df}{dr} = (2x + 2y^2) \cdot 1 + (4xy + 6y^2) \cdot 3 = 2x + 12xy + 20y^2$$

$$\frac{df}{ds} = (2x + 2y^2) \cdot 2 + (4xy + 6y^2) \cdot 2 = 4x + 8xy + 16y^2$$

$$\frac{df}{dr} = 2(r+2s) + 12(r+2s)(3r+2s) + 20(3r+2s)^2$$

$$\frac{df}{ds} = 4(r+2s) + 8(r+2s)(3r+2s) + 16(3r+2s)^2$$

2.) Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if $x^2 + y^2 + z^2 = 5xyz + 1$

$$x^2 + y^2 + z^2 - 5xyz - 1 = 0$$

$$2x + 2z \frac{dz}{dx} - 5y \frac{dz}{dx} - 5yz = 0 \quad 2y + 2z \frac{dz}{dy} - 5xy \frac{dz}{dy} - 5xz = 0$$

$$\frac{dz}{dx} = \frac{-2x + 5yz}{2z - 5y}$$

$$\frac{dz}{dy} = \frac{-2y + 5xz}{2z - 5x}$$