

q9 Rahul Paleja

Section: 22

① Find $\frac{df}{dr}$ and $\frac{df}{ds}$ as functions of r and s , if

$$f(x,y) = x^2 + 2xy^2 + 2y^3$$

$$x = r + 2s \quad y = 3r + 2s \quad \text{No need to simplify}$$

$$\frac{df}{dr} = \left(\frac{df}{dx}\right)\left(\frac{dx}{dr}\right) + \left(\frac{df}{dy}\right)\left(\frac{dy}{dr}\right)$$

$$= \frac{df}{dx} = 2x + 2y^2 \quad \frac{df}{dy} = 4xy + 6y^2$$

$$\frac{dx}{dr} = 1 \quad \frac{dy}{dr} = 3$$

$$\frac{df}{dr} = 2x + 2y^2 + 3(4xy + 6y^2)$$

$$\boxed{\frac{df}{dr} = 12xy + 2x + 20y^2}$$

$$\frac{df}{ds} = \left(\frac{df}{dx}\right)\left(\frac{dx}{ds}\right) + \left(\frac{df}{dy}\right)\left(\frac{dy}{ds}\right)$$

$$\frac{dx}{ds} = 2 \quad \frac{dy}{ds} = 2$$

$$\frac{df}{ds} = 2(2x + 2y^2) + 2(4xy + 6y^2) = 8xy + 4x + 16y^2 = \frac{df}{ds}$$

② Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ $\rightarrow x^2 + y^2 + z^2 = 5xy, z + 1$

W.R.T x : $2x + 2zz' = 5y(xz' + z)$ $u=x \quad v=z$
 $u'=1 \quad v'=z'$

$$2x + 2zz' = 5y(xz' + z) =$$

$$2x + 2zz' = 5xy z' + 5yz$$

$$2zz' - 5xy z' = 5yz - 2x$$

$$z'(2z - 5xy) = 5yz - 2x$$

$$\boxed{\frac{dz}{dx} = \frac{5yz - 2x}{2z - 5xy}}$$

W.R.T y : $2y + 2zz' = 5x(yz' + z)$

$$2y + 2zz' = 5xy z' + 5xz$$

$$2zz' - 5xy z' = 5xz - 2y \rightarrow z'(2z - 5xy) =$$

$$\boxed{\frac{5xz - 2y}{2z - 5xy} = \frac{dz}{dy}}$$