

"QUIZ" for Lecture 9

NAME: (print!) Prathik Lolla Section: _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q9FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 5, 8:00pm

1. Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as functions of r and s , if

$$f(x, y) = x^2 + 2xy^2 + 2y^3,$$

and the variables are related by $x = r + 2s$ and $y = 3r + 2s$. You do not need to simplify!

$$f(x, y) = x^2 + 2xy^2 + 2y^3 \quad x = r + 2s, \quad y = 3r + 2s$$

$$\frac{\partial f}{\partial x} = 2x + 2y^2 \quad \frac{\partial x}{\partial r} = 1 \quad \frac{\partial y}{\partial r} = 3 \quad \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r} = (2x + 2y^2)(1) + (4xy + 6y^2)(3)$$

$$\frac{\partial f}{\partial y} = 4xy + 6y^2 \quad \frac{\partial x}{\partial s} = 2 \quad \frac{\partial y}{\partial s} = 2 \quad \frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} = (2x + 2y^2)(2) + (4xy + 6y^2)(2)$$

$$\frac{\partial f}{\partial r} = (2x + 2y^2)(1) + (4xy + 6y^2)(3)$$

$$\frac{\partial f}{\partial r} = 2x + 2y^2 + 12xy + 18y^2$$

$$\frac{\partial f}{\partial r} = 20y^2 + 12xy + 2x$$

$$\frac{\partial f}{\partial s} = (2x + 2y^2)(2) + (4xy + 6y^2)(2)$$

$$\frac{\partial f}{\partial s} = 4x + 4y^2 + 8xy + 12y^2$$

$$\frac{\partial f}{\partial s} = 16y^2 + 8xy + 4x$$

$$\frac{\partial f}{\partial r} = 20y^2 + 12xy + 2x$$

$$\frac{\partial f}{\partial s} = 16y^2 + 8xy + 4x$$

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$x^2 + y^2 + z^2 = 5xyz + 1$$

Solving for $\frac{\partial z}{\partial x} \Rightarrow \frac{d}{dx}(x^2 + y^2 + z^2 = 5xyz + 1)$

$$2x + 2z \frac{\partial z}{\partial x} = 5xy \frac{\partial z}{\partial x}$$

$$2x = 5xy \frac{\partial z}{\partial x} - 2z \frac{\partial z}{\partial x}$$

$$2x = \frac{\partial z}{\partial x} (5xy - 2z)$$

$$\frac{\partial z}{\partial x} = \frac{2x}{5xy - 2z}$$

Solving for $\frac{\partial z}{\partial y} \Rightarrow \frac{d}{dy}(x^2 + y^2 + z^2 = 5xyz + 1)$

$$2y + 2z \frac{\partial z}{\partial y} = 5xy \frac{\partial z}{\partial y}$$

$$2y = 5xy \frac{\partial z}{\partial y} - 2z \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{5xy - 2z}$$