

$$1. f(x, y) = x^2 + 2xy^2 + 2y^3, \quad x = r + 2s, \quad y = 3r + 2s$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$\frac{\partial f}{\partial x} = 2x + 2y^2, \quad \frac{\partial f}{\partial y} = 4xy + 6y$$

$$\frac{\partial x}{\partial r} = 1, \quad \frac{\partial y}{\partial r} = 3$$

$$\frac{\partial x}{\partial s} = 2, \quad \frac{\partial y}{\partial s} = 2$$

$$\frac{\partial f}{\partial r} = 2x + 2y^2 + 3(4xy + 6y)$$

$$\frac{\partial f}{\partial s} = 2(2x + 2y^2) + 2(4xy + 6y)$$

$$2. \quad x^2 + y^2 + z^2 = 5xyz + 1$$

$$x^2 + y^2 + z^2 - 5xyz - 1 = 0$$

~~$$\frac{d}{dx} = 2x + 2y^2 + z^2 - 5yz -$$~~

$$2x + 2z \frac{dz}{dx} = 5y \frac{d}{dx}(xz)$$

$$2x + 2z \frac{dz}{dx} = 5y \left(x \frac{dz}{dx} + z \right)$$

$$2x + 2z \frac{dz}{dx} = 5xy \frac{dz}{dx} + 5yz$$

$$\frac{dz}{dx} = \frac{5yz - 2x}{2z - 5xy}$$

~~$$\frac{dz}{dx}$$~~

$$\frac{dz}{dy} = \frac{5xz - 2y}{2z - 5xy}$$

