"QUIZ" for Lecture 9

NAME: (print!) Irina Mukhametzhanova **Section:** 24

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q9FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 5, 8:00pm

1. Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as **functions of r and s** , if

$$f(x,y) = x^2 + 2xy^2 + 2y^3 \quad ,$$

and the variables are related by $x = r + 2s$ and $y = 3r + 2s$. You do not need to simplify!

The formula for the chain rule is:

$\frac{\partial f}{\partial r} = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial r}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial r}\right)$ | $\frac{\partial f}{\partial s} = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial s}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial s}\right)$

We first find all of the partial derivatives:

$\frac{\partial f}{\partial x} = 2x + 2y^2$ $\frac{\partial f}{\partial y} = 4xy + 6y^2$ $\frac{\partial x}{\partial r} = 1$ $\frac{\partial x}{\partial s} = 2$ $\frac{\partial y}{\partial r} = 3$ $\frac{\partial y}{\partial s} = 2$

Plug those values into the formulas:

$\frac{\partial f}{\partial r} = (2x + 2y^2)(1) + (4xy + 6y^2)(3)$ | $\frac{\partial f}{\partial s} = (2x + 2y^2)(2) + (4xy + 6y^2)(2)$

$\frac{\partial f}{\partial r} = 2x + 2y^2 + 12xy + 18y^2$ | $\frac{\partial f}{\partial s} = 4x + 4y^2 + 8xy + 12y^2$

$\boxed{\frac{\partial f}{\partial r} = 2x + 20y^2 + 12xy}$ | $\boxed{\frac{\partial f}{\partial s} = 4x + 16y^2 + 8xy}$

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$x^2 + y^2 + z^2 = 5xyz + 1 \quad .$$

We first rearrange the equation to make it implicit:

$x^2 + y^2 + z^2 - 5xyz - 1 = 0$

Now, we take the partial derivatives with respect to x and y:

$2x + 0 + 2zz' - 5y(xz) - 0 = 0$ | $0 + 2y + 2zz' - 5x(yz) - 0 = 0$

$2x + 2zz' - 5yz - 5yxz' = 0$ | $2y + 2zz' - 5xz - 5xyz' = 0$

Factor out z':

$2zz' - 5yxz' = 5yz - 2x$ | $2zz' - 5xyz' = 5xz - 2y$

$z'(2z - 5xy) = 5yz - 2x$ | $z'(2z - 5xy) = 5xz - 2y$

$\frac{\partial z}{\partial x} = \boxed{\frac{5yz - 2x}{2z - 5xy}}$ | $\frac{\partial z}{\partial y} = \boxed{\frac{5xz - 2y}{2z - 5xy}}$