

quiz for lecture 9

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 section 22
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1. Find $\frac{df}{dr}$ and $\frac{df}{ds}$ as function of r and s , if

$$f(x, y) = x^2 + 2xy^2 + zy^3$$

and the variables are related by $x = r + 2s$ and $y = 3r + 2s$.

You do not need to simplify!

① $\frac{df}{dr} = \frac{df}{dx} \frac{dx}{dr} + \frac{df}{dy} \frac{dy}{dr}$

$$\frac{df}{dx} = 2x + 2y^2 \quad \frac{dx}{dr} = 1$$

$$\frac{df}{dy} = 4xy + 6y^2 \quad \frac{dy}{dr} = 3$$

$$\frac{df}{dr} = 2x + 2y^2 + 12xy + 18y^2$$

$$= 2x + 20y^2 + 12xy$$

$$= 2(r + 2s) + 20(3r + 2s)^2 + 12(r + 2s)(3r + 2s)$$

② $\frac{df}{ds} = \frac{df}{dx} \frac{dx}{ds} + \frac{df}{dy} \frac{dy}{ds}$

$$\frac{df}{dx} = 2x + 2y^2 \quad \frac{dx}{ds} = 2$$

$$\frac{df}{dy} = 4xy + 6y^2 \quad \frac{dy}{ds} = 2$$

$$\frac{df}{ds} = 4x + 4y^2 + 8xy + 12y^2$$

$$= 4x + 16y^2 + 8xy$$

$$= 4(r + 2s) + 16(3r + 2s)^2 + 8(r + 2s)(3r + 2s)$$

2. Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if

①

$$x^2 + y^2 + z^2 = 5xyz + 1$$

②

$$2x + 2z z' = 5y(z + xz')$$

$$2x + 2z z' = 5yz + 5xy z'$$

$$(2z - 5xy)z' = 5yz - 2x$$

$$z' = \frac{5yz - 2x}{2z - 5xy}$$

$$\frac{dz}{dx} = \frac{5yz - 2x}{2z - 5xy}$$

③

$$\cancel{*} \quad 2y + 2z z' = 5x(z + yz')$$

$$2y + 2z z' = 5xz + 5xy z'$$

$$(2z - 5xy)z' = 5xz - 2y$$

$$z' = \frac{5xz - 2y}{2z - 5xy}$$

$$\frac{dz}{dy} = \frac{5xz - 2y}{2z - 5xy}$$