Yash Khangura "Quiz" for Lecture 8 Section 24 09/30

1.) Find the directional derivative of the function $f(x,y,z) = xy^2z^3$ at the point $(2,1,1)$ in the direction $\langle 2,-1,-1 \rangle$

$\nabla f(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z) \rangle$

$= \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

$|\langle 2,-1,-1 \rangle| = \sqrt{4+1+1} = \sqrt{6}$

$u = \frac{1}{\sqrt{6}} \langle 2,-1,-1 \rangle = \langle \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \rangle$

$\nabla f(2,1,1) = \langle 1^2 \cdot 1^3, 2 \cdot 2 \cdot 1 \cdot 1^3, 3 \cdot 2 \cdot 1^2 \cdot 1^2 \rangle$

$= \langle 1, 4, 6 \rangle$

$D_u f(x,y,z) = \nabla f(x,y,z) \cdot u = \langle 1,4,6 \rangle \cdot \langle \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \rangle$

$= \frac{2}{\sqrt{6}} + \left(-\frac{4}{\sqrt{6}}\right) + \left(-\frac{6}{\sqrt{6}}\right) = -\frac{8}{\sqrt{6}}$

2.) Find the maximum rate of change of $f(x,y) = x^2 + y^2$ at the point $(2,1)$ and the direction in which is occurs

$\nabla f(x,y) = \langle 2x, 2y \rangle$

$\nabla f(2,1) = \langle 4,2 \rangle$ ← Direction of the maximum rate of change

$|\nabla f(2,1)| = |\langle 4,2 \rangle| = \sqrt{16+4} = \sqrt{20}$ ← maximum rate of change

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