98 Rahul Paleja

Section: 22

① Directional Derivative $f(x,y,z) = xy^2z^3$ at point $(2,1,1)$ in the direction $\langle 2,-1,-1\rangle$

$\nabla f = (f_x, f_y, f_z)$ $\quad f_x = y^2z^3$ $\quad f_y = 2xyz^3$ $\quad f_z = (3xy^2z^2)$

$\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2\rangle$

$|\langle 2,-1,-1\rangle| = \sqrt{2^2+(-1)^2+(-1)^2} = \sqrt{6}$

$u = \langle 2,-1,-1\rangle/\sqrt{6} = \langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle$

$\nabla f(2,1,1) = \langle 1^2 1^3, 2(2)(1)(1)^3, 3(2)(1)^2(1)^2\rangle$

$= \langle 1, 4, 6\rangle$

$\nabla f \cdot u = 1\cdot\frac{2}{\sqrt{6}} + 4\left(\frac{-1}{\sqrt{6}}\right) + 6\cdot\left(\frac{-1}{\sqrt{6}}\right) = \frac{2}{\sqrt{6}} - \frac{4}{\sqrt{6}} - \frac{6}{\sqrt{6}}$

$= \boxed{-\frac{8}{\sqrt{6}} \rightarrow \text{Directional Derivative}}$

② Find maximum rate of change of $f(x,y) = x^2+y^3$ at point $(2,1)$ & the direction it occurs in

$\nabla f = \langle f_x, f_y\rangle$ $\quad f_x = 2x$ $\quad f_y = 3y^2$ $\quad \nabla f = \langle 2x, 3y^2\rangle$

$\nabla f(2,1) = \langle 4, 3\rangle$

Max Rate of Change: $|\langle 4,3\rangle| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

Unit Vector: $\langle 4/5, 3/5\rangle$

Answer: Maximum Rate Of Change is 5 in the direction $\langle 4/5, 3/5\rangle$