

Calc 251 Lecture 8 Quiz (PDF)

1. $f(x, y, z) = xy^2z^3, (2, 1, 1), \langle 2, -1, -1 \rangle$

$$f_x = y^2z^3, f_y = 2xyz^3, f_z = 3xy^2z^2$$

$$\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$$

$$\nabla f(2, 1, 1) = \langle 1, 4, 6 \rangle$$

$$|\langle 2, -1, -1 \rangle| = \sqrt{2^2 + (-1)^2 + (-1)^2} = \sqrt{6}$$

$$\frac{\sqrt{6}}{6} \langle 2, -1, -1 \rangle = \left\langle \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right\rangle$$

$$\langle 1, 4, 6 \rangle \cdot \left\langle \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right\rangle = \frac{\sqrt{6}}{3} - \frac{2\sqrt{6}}{3} - \frac{3\sqrt{6}}{3}$$

The requested directional derivative is $-\frac{4\sqrt{6}}{6}$

2. $f(x, y) = x^2 + y^3, (2, 1)$

$$f_x = 2x, f_y = 3y^2$$

$$\nabla f = \langle 2x, 3y^2 \rangle$$

$$\nabla f(2, 1) = \langle 4, 3 \rangle$$

$$|\langle 4, 3 \rangle| = \sqrt{4^2 + 3^2} = 5$$

The max rate of change is $\left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$