Quiz for lecture 8

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Section: 8:40-10:00 A.M.

1. Find the directional derivative of the function $f(x,y,z)=xy^2z^3$ at the point $(2,1,1)$ in the direction $\langle 2,-1,-1\rangle$.

$$\nabla f=\langle y^2z^3, 2xyz^3, 3xy^2z^2\rangle$$

when the point is $(2,1,1)$

$$\nabla f=\langle 1,4,6\rangle$$

$$\sqrt{2^2+(-1)^2+(-1)^2}=\sqrt{6}$$

the unit vector is $\langle \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}\rangle$

the ~~desired slope~~ directional derivative is $\langle 1,4,6\rangle \cdot \langle \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}\rangle$

$$=\frac{\sqrt{6}}{3}-\frac{2\sqrt{6}}{3}-\sqrt{6}$$

$$=-\frac{4}{3}\sqrt{6}.$$

2. Find the maximum rate of change of $f(x,y)=x^2+y^3$ at point $(2,1)$ and the direction when it occurs.

$$\nabla f=\langle 2x, 3y^2\rangle$$

when the point is $(2,1)$.

$$\nabla f=\langle 4,3\rangle$$

$$D_uf=\nabla f\cdot u=|\nabla f||u|\cos\theta.$$

when $\cos\theta=1$, $\theta=0^\circ$

D_uf maximum.

$$\sqrt{4^2+3^2}=5.$$

$$u=\langle \frac{4}{5}, \frac{3}{5}\rangle.$$

$$D_uf=\langle 4,3\rangle\cdot\langle \frac{4}{5}, \frac{3}{5}\rangle$$

$$=\frac{4}{5}\times 4+\frac{3}{5}\times 3$$

$$=5.$$

And the direction is $5(4i+3j)$