"QUIZ" for Lecture 8

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 1, 2020, 8:00pm

1. Find the directional derivative of the function $f(x, y, z) = xy^2z^3$ at the point $(2, 1, 1)$ in the direction $\langle 2, -1, -1\rangle$.

The formula for the directional derivative at (x_0, y_0, z_0) is:
$\nabla f(x_0, y_0, z_0) \cdot \vec{u}$, when $\vec{u}$ is the unit vector of the direction.
First, we find ∇f:
$\nabla f = \langle f_x, f_y, f_z\rangle = \langle y^2z^3, 2xyz^3, 3xy^2z^2\rangle$
Now, we plug in the point $(2,1,1)$ into the gradient:
$\nabla f(2,1,1) = \langle 1^2 \cdot 1^3, 2(2)(1)(1)^3, 3(2)(1)^2(1)^2\rangle = \langle 1, 4, 6\rangle$
Next, we find the unit vector of $\langle 2,-1,-1\rangle$:
$\vec{u} = \langle 2,-1,-1\rangle / \sqrt{2^2+1^2+1^2} = \langle 2,-1,-1\rangle/\sqrt{6} = \langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle$
Finally, we find the directional derivative:
$\nabla f(2,1,1) \cdot \vec{u} = \langle 1,4,6\rangle \cdot \langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle = \frac{2-4-6}{\sqrt{6}} = \boxed{\frac{-8}{\sqrt{6}}}$

2. Find the maximum rate of change of $f(x, y) = x^2 + y^3$ at the point $(2, 1)$ and the direction in which is occurs.

The maximum rate of change of the function is the magnitude of the gradient at that point:
$\nabla f = \langle f_x, f_y\rangle = \langle 2x, 3y^2\rangle$
$\nabla f(2,1) = \langle 2(2), 3(1)^2\rangle = \langle 4, 3\rangle$
$|\nabla f(2,1)| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$
The direction in which it occurs is the unit vector of this direction:
$\vec{u} = \frac{\nabla f(2,1)}{|\nabla f(2,1)|} = \frac{\langle 4,3\rangle}{5} = \langle \frac{4}{5}, \frac{3}{5}\rangle$
At point $(2,1)$ the maximum rate of change of $f(x,y)$ is $\boxed{5}$ in the direction $\boxed{\langle \frac{4}{5}, \frac{3}{5}\rangle}$