

Quiz 8

1. $f(x, y, z) = xy^2z^3$ at point $(2, 1, 1)$ in direction $\langle 2, -1, -1 \rangle$

$$f_x = y^2z^3 \quad f_y = 2xyz^3 \quad f_z = 3xy^2z^2$$

$$\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$$

$$\sqrt{(2)^2 + (-1)^2 + (-1)^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\text{Unit vector: } \langle 2/\sqrt{6}, -1/\sqrt{6}, -1/\sqrt{6} \rangle$$

$$\text{Plug in to } \nabla f = \langle 1, 4, 6 \rangle$$

$$\begin{aligned} \langle 1, 4, 6 \rangle \cdot \langle 2/\sqrt{6}, -1/\sqrt{6}, -1/\sqrt{6} \rangle \\ = 2/\sqrt{6} + (-4/\sqrt{6}) - 6/\sqrt{6} = \boxed{-8/\sqrt{6}} \end{aligned}$$

2. Max rate of change of $f(x, y) = x^2 + y^3$ at $(2, 1)$ and direction it occurs

$$f_x = 2x \quad f_y = 3y^2$$

$$\nabla f = \langle 2x, 3y^2 \rangle$$

$$\text{At } (2, 1): \nabla f = \langle 4, 3 \rangle$$

$$\text{Max rate of change: } \sqrt{4^2 + 3^2} = \sqrt{16+9} = \boxed{5}$$

$$\text{Direction: } \langle 4/5, 3/5 \rangle$$