NAME: (print!) Afrana Rahman   Section: 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 1, 2020, 8:00pm

1. Find the directional derivative of the function $f(x, y, z) = xy^2z^3$ at the point $(2, 1, 1)$ in the direction $\langle 2, -1, -1 \rangle$.

$\langle f_x, f_y, f_z \rangle \Rightarrow f_x = \frac{\partial f}{\partial x}(xy^2z^3) = y^2z^3$ (y and z are constants)

$\qquad f_y = \frac{\partial f}{\partial y}(xy^2z^3) = (2y)xz^3$ (x and z are constants)

$\qquad f_z = \frac{\partial f}{\partial z}(xy^2z^3) = (3z^2)xy^2$ (x and y are constants)

$|\langle 2, -1, -1 \rangle| = \sqrt{2^2 + 1 + 1} = \sqrt{6} \Rightarrow \frac{1}{\sqrt{6}}$

$u = \frac{1}{\sqrt{6}}\langle 2, -1, -1 \rangle = \langle \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \rangle$

$\nabla f(2,1,1) = \langle (1^2)(1^3), (2(1))(2)(1^3), (3(1^3))(2)(1^2) \rangle = \langle 1, 4, 6 \rangle$

$\nabla f \cdot u = \langle 1, 4, 6 \rangle \cdot \langle \frac{2\sqrt{6}}{6}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \rangle = \frac{2\sqrt{6}}{6} - \frac{4\sqrt{6}}{6} - \frac{6\sqrt{6}}{6} = -\frac{9\sqrt{6}}{6} = \boxed{\frac{3\sqrt{6}}{2}}$

2. Find the maximum rate of change of $f(x, y) = x^2 + y^3$ at the point $(2, 1)$ and the direction in which is occurs.

$\langle f_x, f_y \rangle = \langle 2x, 3y^2 \rangle$

$\nabla f(2,1) = \langle 2(2), 3(1^2) \rangle = \langle 4, 3 \rangle$

$|\langle 4, 3 \rangle| = \sqrt{16 + 9} = 5$

$u = \frac{1}{5}\langle 4, 3 \rangle = \langle \frac{4}{5}, \frac{3}{5} \rangle$

$\boxed{\text{maximum rate of change is 5 in the direction } \langle \frac{4}{5}, \frac{3}{5} \rangle}$