L8: 14.5 pdf Quiz

10/8/20

1. Find the directional deriv. of the fn. $f(x,y,z) = xy^2z^3$ at the pt. $(2,1,1)$ in the direction $\langle 2,-1,-1\rangle$.

$\nabla f = \langle f_x, f_y, f_z\rangle = \langle y^2z^3,\ 2xyz^3,\ 3xy^2z^2\rangle$

$\nabla f_{(2,1,1)} = \langle 1^2\cdot 1^3,\ 2\cdot 2\cdot 1\cdot 1^3,\ 3\cdot 2\cdot 1^2\cdot 1^2\rangle = \langle 1,4,6\rangle$

Gradient

$\|\langle 2,-1,-1\rangle\| = \sqrt{2^2+(-1)^2+(-1)^2} = \sqrt{6}$

$u = \dfrac{\langle 2,-1,-1\rangle}{\sqrt{6}}$

Unit Vector

$\nabla f\cdot u = \langle 1,4,6\rangle \cdot \dfrac{\langle 2,-1,-1\rangle}{\sqrt{6}} = \dfrac{1\cdot 2+4\cdot(-1)+6\cdot(-1)}{\sqrt{6}}$

$= -8/\sqrt{6} = -8\sqrt{6}/6 = -4\sqrt{6}/3$

Directional Deriv.

2. Find the max. rate of Δ of $f(x,y) = x^2+y^3$ at the pt. $(2,1)$ & the direction in which it occurs.

$\nabla f = \langle f_x, f_y\rangle = \langle 2x, 3y^2\rangle$

$\nabla f_{(2,1)} = \langle 2\cdot 2,\ 3\cdot 1^2\rangle = \langle 4,3\rangle$

$\|\nabla f\| = \|\langle 4,3\rangle\| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

Max rate of Δ = magnitude

$\dfrac{\nabla f}{\|\nabla f\|} = \dfrac{\langle 4,3\rangle}{5} = \left\langle \dfrac{4}{5}, \dfrac{3}{5}\right\rangle$

Unit vector