

Quiz 8

Q1. $f(x, y, z) = xy^2z^3$ at $(2, 1, 1)$ direction $(2, -1, -1)$

Answer:

$$\nabla f = (y^2z^3, 2y \cdot xz^3, xy^2 \cdot 3z^2)$$

$$|(2, -1, -1)| = \sqrt{6}$$

$$u = (2, -1, -1) \cdot \frac{1}{\sqrt{6}} = \frac{2\sqrt{6}, -\sqrt{6}, -\sqrt{6}}{6}$$

$$\begin{aligned} u \cdot \nabla f(2, 1, 1) &= \left(\frac{2\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right) \cdot (1, 4, 6) \\ &= \frac{\sqrt{6}}{3} + \left(-\frac{2\sqrt{6}}{3} \right) + (-\sqrt{6}) \\ &= -\frac{\sqrt{6}}{3} - \sqrt{6} \end{aligned}$$

Q2. $f(x, y) = x^2 + y^3$ $(2, 1)$

$$\nabla f = (2x, 3y^2)$$

$$\nabla f = (4, 3)$$

$$|(4, 3)| = \sqrt{4^2 + 3^2} = 5$$

\therefore the maximum rate of change is 5.

