

Yash Khargura "Quiz" for Lecture 6 Section 24

1.) Find the limit if it exists, or show that the limit does not exist

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x}{2x+3y}$$

$$\text{path is } x\text{-axis: } y=0 \rightarrow \lim_{(x,0) \rightarrow (0,0)} \frac{2x}{2x+3(0)} = \frac{2x}{2x} = 1$$

$$\text{path is } y\text{-axis: } x=0 \rightarrow \lim_{(0,y) \rightarrow (0,0)} \frac{2(0)}{2(0)+3y} = \frac{0}{3y} = 0$$

So the limit does not exist

2.) Find the limit if it exists, or show that the limit does not exist

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^5}{x^2+y^2}$$

$$\text{path is } x\text{-axis: } y=0 \rightarrow \lim_{(x,0) \rightarrow (0,0)} \frac{x^5}{x^2+0^2} = \frac{x^5}{x^2} = x^3 = 0$$

$$\text{path is } y\text{-axis: } x=0 \rightarrow \lim_{(0,y) \rightarrow (0,0)} \frac{0^5}{0^2+y^2} = \frac{0}{y^2} = 0$$

$$\text{path is } y=x : \rightarrow \lim_{(x,x) \rightarrow (0,0)} \frac{x^5}{x^2+x^2} = \frac{x^5}{2x^2} = \frac{1}{2}x^3 = 0$$

$$\text{The limit does exist: } \lim_{(x,y) \rightarrow (0,0)} \frac{x^5}{x^2+y^2} = 0$$

$$\text{or } \lim_{(x,y) \rightarrow (0,0)} \frac{x^5}{x^2+y^2}$$

$$\lim_{r \rightarrow 0} \frac{r^5 \cos^5 \theta}{r^2} = r^3 \cos^5 \theta = 0$$

$$\text{The limit does exist: } \lim_{(x,y) \rightarrow (0,0)} \frac{x^5}{x^2+y^2} = 0$$