"QUIZ" for Lecture 6

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q6FirstLast.pdf) ASAP BUT NO LATER THAN Sept. 24, 8:00pm

1. Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2x}{2x+3y}.$$

First, we need to check if the function is continuous at (0,0). If the top or bottom equations vanish, then f(x,y) is not continuous
$2(0)=0$, $2(0)+3(0)=0$ → Results in $\frac{0}{0}$, so, we need to see if that point on the function is approached the same from different directions. We can check if all lines $y=\overset{\text{constant}}{c}x$ that go through (0,0) result in the same limit:

$$\lim_{(x,y)\to(0,0)} \frac{2x}{2x+3y} = \lim_{x\to 0} \frac{2x}{2x+3(cx)} = \lim_{x\to 0} \frac{2x}{(2+3c)x} = \lim_{x\to 0} \frac{2}{2+3c} = \frac{2}{2+3c}$$

Because, for each c-value, the slope changes, the lines do not approach the same limit. Therefore, the limit does not exist

2. Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2}.$$

First, we need to check if the function is continuous at (0,0). If the top or bottom equations vanish, then the checking factors:
$(0)^5=0$, $0^2+0^2=0$ → Results in $\frac{0}{0}$, so, we need to see if that point on the function is approached the same from different directions. We can check if all lines $y=cx$ that go through $y=\overset{\text{constant}}{c}x$ that go through (0,0) result in the same limit:

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2} = \lim_{x\to 0} \frac{x^5}{x^2+(cx)^2} = \lim_{x\to 0} \frac{x^5}{x^2+c^2x^2} = \lim_{x\to 0} \frac{x^5}{(1+c^2)x^2} = \lim_{x\to 0} \frac{x^3}{1+c^2} = 0$$

So, 0 could be the limit, but we can check if it is correct by converting to polar coordinates ($x=r\cos\theta$, $y=r\sin\theta$):

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2} = \lim_{r\to 0} \frac{r^5\cos^5\theta}{r^2(\cos^2\theta+\sin^2\theta)} = \lim_{r\to 0} r^3\cos^5\theta = 0 \checkmark \quad 0=0, \text{ so, the limit of the function at } (0,0) \text{ is } 0$$