L6: 14.1, 14.2 pdf Quiz

10/3/20

1. Find the limit if it exists, or show that the limit DNE.

$$\lim_{(x,y)\to(0,0)} \frac{2x}{2x+3y} = \frac{\text{"0"}}{0} \quad \text{(INDETERMINATE)}$$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=mx}} \frac{2x}{2x+3y} = \lim_{x\to 0} \frac{2x}{2x+3mx} = \lim_{x\to 0} \frac{2x}{(2+3m)x} = \lim_{x\to 0} \frac{2}{(2+3m)}$$

$$= \frac{2}{(2+3m)} \quad \left(\text{Depends on } m \Rightarrow \text{limit DNE}\right)$$

2. Find the limit if it exists, or show that the limit DNE.

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2} = \frac{\text{"0"}}{0} \quad \text{(INDETERMINATE)}$$

If we do as above, we would get that the limit is 0, no matter on which $y=mx$ you are traveling to $(0,0)$. So the limit <u>probably</u> exists, & if it does it is = to 0. But this is <u>not</u> a conclusive proof. To prove it conclusively, we convert to polar:

$$x = r\cos\theta \quad , \quad y = r\cos\theta$$

Recall that $x^2+y^2=r^2$, so

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2} = \lim_{r\to 0} \frac{(r\cos\theta)^5}{r^2} = \lim_{r\to 0} \frac{r^5\cos^5\theta}{r^2}$$

$$= \lim_{r\to 0} r^3\cos^5\theta = \cos^5\theta \lim_{r\to 0} r^3$$

$$= \cos^5\theta \cdot 0 = 0$$