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"Quiz" for Lecture 5

Section 24

1.) Find the curvature for $r(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + \mathbf{k}$

$$r'(t) = \langle \cos t, -\sin t, 0 \rangle$$
$$r''(t) = \langle -\sin t, -\cos t, 0 \rangle$$
$$r'(t) \times r''(t) = \langle 0, 0, -(\cos^2 t + \sin^2 t) \rangle = \langle 0, 0, -1 \rangle$$
$$|r'(t) \times r''(t)| = |\langle 0, 0, -1 \rangle| = \sqrt{1} = 1$$
$$|r'(t)| = \sqrt{\cos^2 t + \sin^2 t + 0} = \sqrt{1} = 1$$
$$\frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{1}{(1)^3} = 1 = K(t)$$

2.) Find the velocity, acceleration, and speed of a particle with the given position function $r(t) = t \mathbf{i} + t^2 \mathbf{j} + 5 \mathbf{k}$

$$v(t) = r'(t) = \langle 1, 2t, 0 \rangle$$
$$a(t) = r''(t) = \langle 0, 2, 0 \rangle$$
$$\text{Speed} = |v(t)| = \sqrt{1^2 + (2t)^2 + 0^2} = \sqrt{1 + 4t^2}$$