q5 Rahul Paleja

Section: 22

① Find the curvature for

$r(t) = \sin t\, i + \cos t\, j + t\, K$

$r'(t) = \cos t\, i - \sin t\, j + K \rightarrow \langle \cos t, -\sin t, 1 \rangle$

$r''(t) = -\sin t\, i - \cos t\, j \rightarrow \langle -\sin t, -\cos t, 0 \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ \cos t & -\sin t & 1 \\ -\sin t & -\cos t & 0 \end{vmatrix} = i(\cos t) - j(\sin t) + K(-\cos^2 t - \sin^2 t)$$

$$= i(\cos t) - j(\sin t) + K(-(\cos^2 t + \sin^2 t))$$

$$= i(\cos t) - j(\sin t) + K(-1)$$

$$= \langle \cos t, -\sin t, -1 \rangle$$

$$|r'(t) \times r''(t)| = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{1+1} = \sqrt{2}$$

$$|r'(t)| = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{1+1} = \sqrt{2}$$

$$= \frac{\sqrt{2}}{(\sqrt{2})^3} = \frac{2^{1/2}}{2^{3/2}} = \frac{1}{2^{2/2}} = \frac{1}{2^1} = \boxed{\frac{1}{2}} = K(t)$$

② Find the velocity, acceleration, and speed of particle with position:

$r(t) = t\, i + t^2\, j + 5k$

$v(t) = r'(t) = 1\, i + 2t\, j + 0K = \langle 1, 2t, 0 \rangle$

$a(t) = r''(t) = 0\, i + 2\, j + 0K = \langle 0, 2, 0 \rangle$

Speed = Magnitude of velocity $= |v(t)| = \sqrt{1^2 + (2t)^2 + 0^2}$

$$= \boxed{\sqrt{4t^2 + 1}}$$