

"QUIZ" for Lecture 5

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q5FirstLast.pdf) ASAP BUT NO LATER THAN Sept. 21, 8:00pm

1, Find the curvature for

$$r(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k}$$

The formula for curvature is $\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

First, we need to find $r'(t)$ and $r''(t)$:

$$r'(t) = (\sin t)' \mathbf{i} + (\cos t)' \mathbf{j} + (t)' \mathbf{k} = \cos t \mathbf{i} - \sin t \mathbf{j} + \mathbf{k}$$

$$r''(t) = (\cos t)' \mathbf{i} + (-\sin t)' \mathbf{j} + (1)' \mathbf{k} = -\sin t \mathbf{i} - \cos t \mathbf{j}$$

We find the cross product of $r'(t)$ and $r''(t)$:

$$r'(t) \times r''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos t & -\sin t & 1 \\ -\sin t & -\cos t & 0 \end{vmatrix} = \mathbf{i}(-\sin t \cdot 0 - (-\cos t)(1)) - \mathbf{j}(\cos t \cdot 0 - (-\sin t)(1)) + \mathbf{k}(\cos t)^2 - (\sin t)^2$$

$$= \mathbf{i}(\cos t) - \mathbf{j}(\sin t) + \mathbf{k}(-1) = \cos t \mathbf{i} - \sin t \mathbf{j} - \mathbf{k}$$

$$|r'(t) \times r''(t)| = \sqrt{\cos^2 t + \sin^2 t + (-1)^2} = \sqrt{2} \quad \rightarrow |r'(t)|^3 = (\sqrt{2})^3 = 2\sqrt{2}$$

$$|r'(t)| = \sqrt{\cos^2 t + (-\sin t)^2 + 1} = \sqrt{2} \quad \kappa(t) = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$$

2.: Find the velocity, acceleration, and speed of a particle with the given position function.

$$r(t) = t \mathbf{i} + t^2 \mathbf{j} + 5 \mathbf{k}$$

The velocity is the derivative of the position function:

$$v(t) = r'(t) = (t)' \mathbf{i} + (t^2)' \mathbf{j} + (5)' \mathbf{k} = \mathbf{i} + 2t \mathbf{j}$$

The acceleration is the derivative of the velocity function:

$$a(t) = v'(t) = (1)' \mathbf{i} + (2t)' \mathbf{j} = 2 \mathbf{j}$$

The speed is the magnitude of the velocity vector:

$$\text{speed} = |v(t)| = \sqrt{1^2 + (2t)^2} = \sqrt{1+4t^2}$$