

“QUIZ” for Lecture 5

NAME: (print!) Fady Besada

Section: 22

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q5FirstLast.pdf) ASAP BUT NO LATER THAN Sept. 21, 8:00pm

1. Find the curvature for

$$\rightarrow \mathbf{r}(t) = \langle \sin(t), \cos(t), t \rangle \quad \mathbf{r}(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k}$$

$$\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\rightarrow \mathbf{r}'(t) = \langle \cos(t), -\sin(t), 1 \rangle$$

$$\rightarrow \mathbf{r}''(t) = \langle -\sin(t), -\cos(t), 0 \rangle$$

$$\rightarrow \mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos t & -\sin t & 1 \\ -\sin t & -\cos t & 0 \end{vmatrix} = (-\cos t) \mathbf{i} - (\sin t) \mathbf{j} + (-\cos^2 t - \sin^2 t) \mathbf{k}$$

$$\rightarrow |-\cos t \cdot \mathbf{i} - \sin t \cdot \mathbf{j} - \mathbf{k}| = |<-\cos t, -\sin t, -1>| = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2}$$

$$\rightarrow |\langle \cos(t), -\sin(t), 1 \rangle|^3 = (\sqrt{\cos^2 t + \sin^2 t + 1})^3 = (\sqrt{2})^3 = 2\sqrt{2} \rightarrow \frac{\sqrt{2}}{2\sqrt{2}} \rightarrow \boxed{\text{Ans: } \frac{1}{2}}$$

2.: Find the velocity, acceleration, and speed of a particle with the given position function.

$$\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + 5 \mathbf{k}$$

$$\rightarrow \mathbf{v}(t) = \mathbf{r}'(t) = \langle 1, 2t, 0 \rangle$$

$$\rightarrow \boxed{\mathbf{v}(t) = \mathbf{i} + 2t \mathbf{j}} \leftarrow \text{velocity}$$

$$\rightarrow \mathbf{a}(t) = \mathbf{r}''(t) = \langle 0, 2, 0 \rangle$$

$$\rightarrow \boxed{\mathbf{a}(t) = 2 \mathbf{j}} \leftarrow \text{acceleration}$$

$$\rightarrow s(t) = |\mathbf{v}(t)| = \sqrt{1 + 4t^2}$$

$$\rightarrow \boxed{s(t) = \sqrt{1+4t^2}} \leftarrow \text{speed}$$