

"QUIZ" for Lecture 5

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q5FirstLast.pdf) ASAP BUT NO LATER THAN Sept. 21, 8:00pm

1, Find the curvature for

$$\begin{aligned}
 \mathbf{r}(t) &= \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k} = \langle \sin t, \cos t, t \rangle \\
 \mathbf{r}'(t) &= \langle \cos t, -\sin t, 1 \rangle \\
 \mathbf{r}''(t) &= \langle -\sin t, -\cos t, 0 \rangle
 \end{aligned}$$

$$\begin{aligned}
 \kappa &= \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} \\
 &= \frac{|\langle \cos t, -\sin t, 1 \rangle \times \langle -\sin t, -\cos t, 0 \rangle|}{|\langle \cos t, -\sin t, 1 \rangle|^3} = \frac{|\langle 0 + \cos t, -\sin t - 0, -\cos^2 t - \sin^2 t \rangle|}{|\langle \cos t, -\sin t, 1 \rangle|^3} \\
 &= \frac{|\langle \cos t, -\sin t, -1 \rangle|}{|\langle \cos t, -\sin t, 1 \rangle|^3} = \frac{\sqrt{\cos^2 t + \sin^2 t + 1}}{(\sqrt{\cos^2 t + \sin^2 t + 1})^3} = \frac{\sqrt{2}}{\sqrt{2}^3} = \frac{1}{\sqrt{2}^2} = \left(\frac{1}{2}\right)
 \end{aligned}$$

2.: Find the velocity, acceleration, and speed of a particle with the given position function.

$$\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + 5 \mathbf{k} = \langle t, t^2, 5 \rangle$$

$$\text{velocity} = \mathbf{v}(t) = \mathbf{r}'(t) = \langle 1, 2t, 0 \rangle$$

$$\text{speed} = |\mathbf{v}(t)| = |\mathbf{r}'(t)| = \sqrt{1^2 + (2t)^2} = \sqrt{1 + 4t^2}$$

$$\text{acceleration} = \mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = \langle 0, 2, 0 \rangle$$