

1. Find the curvature for $r(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k}$

$$r(t) = \langle \sin t, \cos t, t \rangle$$

$$\text{curvature formula: } \kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$r'(t) = \langle \cos t, -\sin t, 1 \rangle$$

$$r''(t) = \langle -\sin t, -\cos t, 0 \rangle$$

$$\begin{aligned} r'(t) \times r''(t) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos t & -\sin t & 1 \\ -\sin t & -\cos t & 0 \end{vmatrix} \\ &= \mathbf{j} \begin{vmatrix} -\sin t & 1 \\ -\cos t & 0 \end{vmatrix} - \mathbf{j} \begin{vmatrix} \cos t & 1 \\ -\sin t & 0 \end{vmatrix} + \mathbf{k} \begin{vmatrix} \cos t & -\sin t \\ -\sin t & -\cos t \end{vmatrix} \\ &= \mathbf{i} \cos t - \mathbf{j}(\sin t) + \mathbf{k}(-\cos^2 t - \sin^2 t) \\ &= (\cos t) \mathbf{i} + (-\sin t) \mathbf{j} + (-1) \mathbf{k} \end{aligned}$$

$(\cos^2 t + \sin^2 t = 1)$

$$r'(t) \times r''(t) = \langle \cos t, -\sin t, -1 \rangle$$

$$\begin{aligned} \text{Magnitude: } |r'(t) \times r''(t)| &= \sqrt{(\cos t)^2 + (-\sin t)^2 + (-1)^2} \\ &= \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2} \end{aligned}$$

$$|r'(t)| = |\langle \cos t, -\sin t, 1 \rangle| = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2}$$

$$\kappa(t) = \frac{\sqrt{2}}{\sqrt{2}^3} = \frac{1}{2}$$

2. Find the velocity, acceleration, & speed of a particle w/ given position fn:

$$r(t) = t \mathbf{i} + t^2 \mathbf{j} + 5 \mathbf{k}$$

$$v(t) = t' \mathbf{i} + (t^2)' \mathbf{j} + 5' \mathbf{k} = 1 \mathbf{i} + (2t) \mathbf{j} + 0 \mathbf{k} = \mathbf{i} + 2t \mathbf{j}$$

$$a(t) = 1' \mathbf{i} + (2t)' \mathbf{j} = 0 \mathbf{i} + 2 \mathbf{j} = 2 \mathbf{j}$$

$$\text{Speed: } |v(t)| = \sqrt{1^2 + (2t)^2} = \sqrt{1 + 4t^2}$$