

Quiz for lecture 5

$$Q1: \text{Given } r(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k}$$

$$r'(t) = \cos t \mathbf{i} + (-\sin t) \mathbf{j} + (1) \mathbf{k} = (\cos t, -\sin t, 1)$$

$$r''(t) = (-\sin t) \mathbf{i} + (-\cos t) \mathbf{j} + 0 = (-\sin t, -\cos t, 0)$$

$$r'(t) \times r''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos t & -\sin t & 1 \\ -\sin t & -\cos t & 0 \end{vmatrix}$$

$$= (\cos^2 t \mathbf{i} + \sin^2 t \mathbf{j} + (-\cos^2 t - \sin^2 t) \mathbf{k})$$

$$\|r'(t) \times r''(t)\| = \sqrt{\cos^2 t + \sin^2 t + (-\cos^2 t - \sin^2 t)^2}$$

$$\|r'(t)\| = \sqrt{\cos^2 t + (-\sin t)^2 + 1}$$

$$k(t) = \frac{\sqrt{\cos^2 t + \sin^2 t + \cos^4 t + \sin^4 t + 2\sin^2 t \cos^2 t}}{\sqrt{\cos^2 t + \sin^2 t + 1}}$$

$$Q2: r(t) = t \mathbf{i} + t^2 \mathbf{j} + 5t \mathbf{k}$$

$$\text{velocity} = r'(t) = \mathbf{i} + 2t \mathbf{j} + 5 \mathbf{k} = (1, 2t, 5)$$

$$\text{acceleration} = 0 \mathbf{i} + 2 \mathbf{j} + 0 \mathbf{k} = (0, 2, 0)$$

$$\text{speed} = \|r'(t)\| = \sqrt{1^2 + 4t^2 + 25} = \sqrt{6 + 4t^2}$$

