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Quiz for Lecture 5

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section 22.

Find the curvature for

$$r(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k}$$

$$\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$r'(t) = \cos t \mathbf{i} - \sin t \mathbf{j} + \mathbf{k} = \langle \cos t, -\sin t, 1 \rangle$$

$$r''(t) = -\sin t \mathbf{i} - \cos t \mathbf{j} = \langle -\sin t, -\cos t, 0 \rangle$$

$$\begin{aligned} \kappa &= r'(t) \times r''(t) = \langle \cos t, -\sin t, 1 \rangle \times \langle -\sin t, -\cos t, 0 \rangle \\ &= \langle \cos t, -\sin t, -\cos^2 t - \sin^2 t \rangle \end{aligned}$$

$$\begin{aligned} |r'(t) \times r''(t)| &= \sqrt{\cos^2 t + \sin^2 t + (\cos^2 t)^2 + 2(\cos t \sin t)^2 + (\sin^2 t)^2} \\ &= \sqrt{1 + [\cos^2 t + \sin^2 t]^2} \\ &= \sqrt{1 + 1^2} \\ &= \sqrt{2} \end{aligned}$$

$$|r'(t)| = \sqrt{\cos^2 t + \sin^2 t + 1}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{\sqrt{2}}{(\sqrt{2})^3} = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$$

2. Find the velocity, acceleration, and speed of a particle with the given position function.

$$r(t) = t\mathbf{i} + t^2\mathbf{j} + 5\mathbf{k}.$$

$$v(t) = r'(t) = \mathbf{i} + 2t\mathbf{j} \neq \mathbf{k} = \langle 1, 2t, 0 \rangle$$

$$a(t) = r''(t) = v'(t) = 2\mathbf{j} = \langle 0, 2, 0 \rangle$$

$$\text{speed} = |v(t)| = \sqrt{1^2 + (2t)^2 + 0^2} = \sqrt{1 + 4t^2}.$$