

"QUIZ" for Lecture 4

NAME: (print!) Irina Mukhametzhanova Section: 24

E-MAILSCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q4FirstLast.pdf) ASAP BUT NO LATER THAN Sept. 17, 8:00pm

1. Find a parametric equation for the tangent line to the curve with the given parametric equation at the specified point

$$x = \cos t, \quad y = \sin t, \quad z = t^2 + 1; \quad (1, 0, 1)$$

We find the tangent line by finding the derivative of the equations of the curve (or of its $x, y,$ and z components)

$$r'(t) = \langle (\cos t)', (\sin t)', (t^2 + 1)' \rangle = \langle -\sin t, \cos t, 2t \rangle$$

We need to find the t -value where $r(t) = \langle 1, 0, 1 \rangle$:

$$\cos t = 1, \quad \sin t = 0, \quad t^2 + 1 = 1 \rightarrow t = 0$$

Find the tangent vector at $t = 0$:

$$r'(0) = \langle -\sin 0, \cos 0, 2 \cdot 0 \rangle = \langle 0, 1, 0 \rangle$$

The equation for the tangent line is $(1, 0, 1) + t \langle 0, 1, 0 \rangle =$

$$= \langle 1 + 0t, 0 + 1t, 1 + 0t \rangle = \langle 1, t, 1 \rangle; \quad \text{or } \boxed{x=1 \quad y=t \quad z=1}$$

2. Find $r(t)$ if

$$r'(t) = t\mathbf{i} + 2\mathbf{j} + (t+1)\mathbf{k}$$

and

$$r(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

To find $r(t)$, we first need to integrate $r'(t)$

$$r(t) = \int r'(t) dt = \frac{t^2}{2} \mathbf{i} + 2t \mathbf{j} + \left(\frac{t^2}{2} + t \right) \mathbf{k} + C$$

To find C , use a value we already know:

$$r(0) = \frac{0^2}{2} \mathbf{i} + 2(0) \mathbf{j} + \left(\frac{0^2}{2} + 0 \right) \mathbf{k} + C = C = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

So, $r(t)$ is:

$$r(t) = \frac{t^2}{2} \mathbf{i} + 2t \mathbf{j} + \left(\frac{t^2}{2} + t \right) \mathbf{k} + \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

$$r(t) = \left[\left(\frac{t^2}{2} + 1 \right) \mathbf{i} + (2t + 2) \mathbf{j} + \left(\frac{t^2}{2} + t + 3 \right) \mathbf{k} \right]$$