L4 : 13.1, 13.2 pdf Quiz

10/3/20

1. Find a parametric eq. for the tangent line to the curve w/ the given parametric eq. at the specified pt.

$x = \cos t$, $y = \sin t$, $z = t^2 + 1$; $(1, 0, 1)$ (pt.)

vector form : $r(t) = \langle \cos t, \sin t, t^2 + 1 \rangle$

$r'(t) = \langle -\sin t, \cos t, 2t \rangle$

Decide what time, t, it is when particle is at $(1, 0, 1)$

$t^2 + 1 = 1 \Rightarrow t = 0$

$r'(0) = \langle -\sin 0, \cos 0, 2 \cdot 0 \rangle = \langle 0, 1, 0 \rangle$ (direction of tangent)

eq. of tangent line : $\langle 1, 0, 1 \rangle + t\langle 0, 1, 0 \rangle = \langle 1, t, 1 \rangle$

scalar form $\Rightarrow x = 1$, $y = t$, $z = 1$

The parametric eq. of the tangent line to the given curve at the specified pt. is $x = 1$, $y = t$, $z = 1$ $(-\infty < t < \infty)$

2. Find $r(t)$ if $r'(t) = ti + 2j + (t+1)K$ & $r(0) = i + 2j + 3K$

$$r(t) = \int (ti + 2j + (t+1)K)\, dt = \frac{t^2}{2} i + 2tj + \left(\frac{t^2}{2} + t\right) K + C$$

Find what vector C is $\Rightarrow$ plug-in $t = 0$ into $r(t)$

$r(0) = \cancel{0i + 0j + 0K + C} = C$

$C = i + 2j + 3K$

$$r(t) = \frac{t^2}{2} i + 2tj + \left(\frac{t^2}{2} + t\right) K + i + 2j + 3K$$

$$= \left(\frac{t^2}{2} + 1\right) i + (2t + 2) j + \left(\frac{t^2}{2} + t + 3\right) K$$

$$\approx \left\langle \frac{t^2}{2} + 1 , 2t + 2 , \frac{t^2}{2} + t + 3 \right\rangle$$