L3: 12.5 pdf Quiz

10/3/20

1. Find an eq. of the plane that passes through the pts: $(0,1,1)$, $(1,0,1)$, $(1,1,0)$

Let $P=(0,1,1)$, $Q=(1,0,1)$, $R=(1,1,0)$

Need 2 direction vectors that lie on the plane:

$$PQ = \langle 1-0, 0-1, 1-1\rangle = \langle 1,-1,0\rangle$$

$$PR = \langle 1,0,-1\rangle$$

To get the normal, cross-product $PQ \times PR$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = i\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$

$$= i((-1)\cdot(-1)-0\cdot 0) - j(1\cdot(-1)-0\cdot 1) + k(1\cdot 0-(-1)\cdot 1)$$

$$= i+j+k$$

Convert $\Rightarrow \langle 1,1,1\rangle = n$

Eq. of general plane: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$\langle a,b,c\rangle = \langle 1,1,1\rangle$, (x_0,y_0,z_0) = any pt. (P, Q, or R)

$$1(x-0)+1(y-1)+1(z-1)=0$$

$$x+y+z=2$$

2. Find the intersection of the line $r(t)=\langle 1,1,0\rangle + t\langle 0,2,4\rangle$ & the plane $x+y+z=14$

$r(t) = \langle 1,1,0\rangle + t\langle 0,2,4\rangle = \langle 1, 1+2t, 4t\rangle$

scalar form $\Rightarrow$ $x=1$, $y=1+2t$, $z=4t$

$1+(1+2t)+4t=14 \Rightarrow 2+6t=14 \Rightarrow 6t=12 \Rightarrow t=2$

$x=1$, $y=1+2\cdot 2=5$, $z=4\cdot 2=8$

The intersection of the line & the plane given by the problem is the pt. $(1,5,8)$.