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2017 final c pdf quiz 267

Q1. $\int_C 7y\,dx + 3x\,dy$

$C \Rightarrow x^2 + y^2 = 100$ clockwise.

$P = -y^3 \quad Q = x^3$

$Q_{-x} - P_{-y} = 3(x^2+y^2)$

$\int_0^{2\pi}\int_0^5 3r^2 \cdot r\,dr\,d\theta = 1250\pi$

Ans: $\cancel{r = 0..10 \quad \theta = 0..2\pi}$

$y = \sqrt{100 - x^2}$

$\int_0^{2\pi}\int_0^{10} (7r\sin\theta + 3\cdot r\cos\theta)\, r\,dr\,d\theta$

$= \int_0^{\pi} \frac{7}{3}r^3\sin\theta + r^3\cos\theta \Big|_0^{10} d\theta$

$= \int_0^{\pi} \frac{7000}{3}\sin\theta + 1000\cos\theta \; d\theta = \frac{7000}{3} + 1000\sin\pi - \frac{7000\cos\theta}{3}$

Q2. $z = x^2 + 3xy + y^2 \quad (1,1,5)$

Ans: $f_{-x} = 2x + 3y = 5$

$f_{-y} = 3x + 2y = 5$

$z - 5 = 5(x-1) + 5(y-1)$

$z = 5x + 5y - 5$

Q3. $f(x,y) = x^2y \quad \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1-x\}$

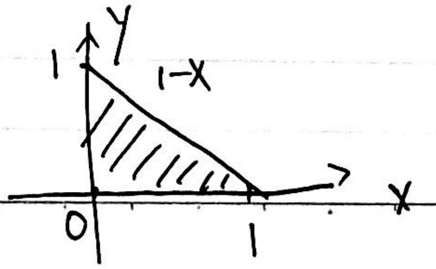
Ans: $f_x = 2xy \quad f_y = x^2$

$2xy = 0 \quad x^2 = 0$

$x = 0$ or $y = 0 \quad \therefore (0,0) \quad f(0,0) = 0$

$\cancel{① \ x = 0,\ y = 0..1-x}$

$\cancel{f(0,y) = 0}$



① $x = 0 \quad y = 0..1$

② $x = 0..1 \quad y = 0..1-x$

③ $x = 0..1 \quad y = 0$

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$f(x,y)=x^2y$

① when $y=0$, $x=0$ $f=0$

$x=1$ $f=0$

② when $x=0$, $y=0$ $f=0$

$y=1$ $f=0$

③ when $x=0$ $y=0$ $f=0$

$x=1$, $y=0$ $f=0$

$x=1$ $y=1-x$ $f=1-x$

$\therefore$ the maximum is $1-x$, minimum is 0.

Q4. $f(x,y,z)=\sin(x^2+y+z)$

$f_x = 2x\cdot\cos(x^2+y+z)$

$f_{xx} = 2\cos(x^2+y+z)+(2x)^2\cdot(-\sin(x^2+y+z))$

$= 2\cos(x^2+y+z)+4x^2\cdot(-\sin(x^2+y+z))$

$f_{xxy} = \cancel{-2\sin(x^2+y+z)}+(-4x^2\cdot\cancel{\sin}\cos(x^2+y+z))$

$f_{xxyz} = -2\cos(x^2+y+z)+4x^2\cdot\sin(x^2+y+z)$

plug in $(0,0,0)$ $= -2\cos 0+0$

$= -2$

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Q5. $\frac{dz}{dy}$: $xy + xz + yz + x^2y^2z^2 = 4$

$$0 = x + x\cdot\frac{dz}{dy} + z + y\cdot\frac{dz}{dy} + x^2\cdot 2y\cdot z^2 + 2z\cdot\frac{dz}{dy}\cdot x^2y^2$$

$$0 = \frac{dz}{dy}(x + y + 2z\cdot x^2y^2) + x + z + 2x^2yz^2$$

$$\frac{dz}{dy}(1,1,1) = \frac{-x-z-2x^2yz^2}{x+y+2z\cdot x^2y^2} = \frac{-2-2}{2+2} = \frac{-4}{4} = -1$$

Q6. line a: $(1+t, 2+t, 3+t)$

b: $(-t, 1+t, 2+t)$

$$a\times b = \begin{vmatrix} 1+t & 2+t & 3+t \\ -t & 1+t & 2+t \end{vmatrix} = \left((2+t)^2 - (1+t)\cdot(3+t)\right)i - \left[(1+t)\cdot(2+t) + t\cdot(3+t)\right]j + \left[(1+t)^2 + (2+t)\cdot t\right]k$$

$$= (1, -2t^2-6t-2, 2t^2+4t+1)$$

$$0 = (x+t) + (-2t^2-6t-2)\cdot(y-1-t) + (2t^2+4t+1)\cdot(z-2-t)$$

$$z = 2y - x - (-2y+2z)t^2 - (-6y+4z)t$$

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Q8.7.

$$\int a(t)\,dt = v(t) = \int -4\sin 2t + (-4\cos 2t) + 9e^{3t}\,dt$$

$$= 2\cos(2t)i - 2\sin(2t)j + 3e^{3t}k + C$$

$v(0) = (2, 0, 3)$

$\therefore C = 0$

$v(t) = 2\cos(2t)i - 2\sin(2t)j + 3e^{3t}k$

$r(t) = \int v(t)\,dt = \sin(2t)i + \cos(2t) + e^{3t} + C$

$r(0) = (0, 1, 1)$

$\therefore C = 0$

$r(t) = \sin(2t)i + \cos(2t)j + e^{3t}k$

$r(\frac{\pi}{4}) = (1, 0, e^{\frac{3\pi}{4}})$

Q8.8. $\int_C (x+y+2z)\,ds \quad r(t) = (t, 2t, 2t) \quad 0 \le t \le 1$

Ans: $ds = \sqrt{1+4+4} = 3dt$, $r'(t) = (1, 2, 2)$

$$\int_0^1 (t + 2t + 4t)\cdot 3\,dt$$

$$= \int_0^1 21t\,dt$$

$$= \frac{21t^2}{2}\Big|_0^1$$

$$= \frac{21}{2} = 10.5$$

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Q9.

$$\lim_{(x,y,z)\to(1,1,1)} \sin\left(\frac{\pi}{3} f(x,y,z)\right)\cos\left(\frac{\pi}{4} g(x,y,z)\right)$$

$$= \sin\left(\frac{\pi}{3}\cdot 1\right)\cos\left(\frac{\pi}{4}\cdot 2\right)$$

$$= \sin\frac{\pi}{3}\cdot\cos\frac{\pi}{2}$$

$$= 0$$

Q10. $\iint_S F\cdot dS$

$$= \iint_D \left(-P\frac{dz}{dx} - Q\frac{dz}{dy} + R\right) dA$$

$$\frac{dz}{dx} = y\cdot e^{xy} \quad \frac{dz}{dy} = x\cdot e^{xy}$$

$$= \int_0^1\int_0^y \left(x^2+\sin(y+z)\right)\cdot y\cdot e^{xy} - (y^2+xz^3)\cdot x\cdot e^{xy} + (z^2+e^{xy})\ dx\,dy.$$

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Q11. $\cancel{F(x,y,z)=(2e^{2t+6t+4t^2}, 3e^{8t+4t^2}, 4e^{8t+4t^2})}$

$$\int f_x \, dx = e^{2x+3y+4z} + C$$

$$\frac{d}{dy}\cdot f = 3e^{2x+3y+4z} \qquad \frac{d}{dz}\cdot f = 4e^{2x+3y+4z}$$

$$\therefore C = 0$$

$$\therefore f = \int_C F\cdot dr = e^{2x+3y+4z}$$

Q12. $\int_C 5y\,dx + 5x\,dy + 6z\,dz$

$x=t^2,\ y=t\ \ z=t^2 \quad 0\le t\le 1$

Ans: $\int_0^1 5(t)\cdot 2t\,dt + 5(t^2)\,dt + 6t^2\cdot 2t\,dt$

$$= \int_0^1 10t^2 + 5t^2 + 12t^3\,dt$$

$$= \int_0^1 15t^2 + 12t^3\,dt$$

$$= 5t^3 + 3t^4 \Big|_0^1$$

$$= 8$$

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Q13. $\iiint_E \frac{1}{\sqrt{x^2+y^2+z^2}} \, dv$

$\{(x,y,z) \mid x^2+y^2+z^2 \le 100, \; z<0\}$

Ans: $x^2+y^2+z^2 = R^2$

$\iiint_E \frac{1}{R} \, dv$

$= \int_0^{\pi}\int_0^{2\pi}\int_0^{10} \frac{1}{\rho} \cdot \rho^2 \sin\phi \, d\rho \cdot d\theta \, d\phi$

$= \int_0^{\pi}\int_0^{2\pi}\int_0^{10} \rho \cdot \sin\phi \, d\rho \, d\theta \, d\phi$

$= \int_0^{\pi}\int_0^{2\pi} \frac{\rho^2}{2} \sin\phi \Big|_0^{10} \, d\theta \, d\phi$

$= \int_0^{\pi}\int_0^{2\pi} 50 \sin\phi \, d\theta \, d\phi$

$= \int_0^{\pi} 100\pi \cdot \sin\phi \, d\phi$

$= 100\pi \cdot -\cos\phi \Big|_0^{\pi}$

$= 100\pi (1+1)$

$= 200\pi.$

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Q14.

$$\int_0^1 \int_0^{\omega} \int_0^{z} \int_0^{y} 360x \, dx\, dy\, dz\, dw$$

$$= \int_0^1 \int_0^{w} \int_0^{z} 180x^2 \Big|_0^{y} \, dy\, dz\, dw$$

$$= \int_0^1 \int_0^{\omega} \int_0^{z} 180y^2 \, dy\, dz\, dw$$

$$= \int_0^1 \int_0^{\omega} 60y^3 \Big|_0^{z} \, dz\, dw$$

$$= \int_0^1 15z^4 \Big|_0^{\omega} \, dw$$

$$= \int_0^1 15\omega^4 \, dw$$

$$= 3\omega^5 \Big|_0^1$$

$$= 3$$

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Q15.

Jacobian = $\cancel{6\sin\cos(2u+v)\cdot[(1+-\sin(u+v))}$

$$\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = 6\cos(2u+v)\cdot(1+-\sin(u+v)) - (1-\sin(u+v))\cdot 3(\cos(2u+v))$$

$$=(1-\sin(u+v))\cdot(6\cos(2u+v) - 3\cos(2u+v))$$

$$=(1-\sin 0)\cdot(3\cos 0)$$

$$= 1\cdot 3 = 3$$

Q16. $f(x,y) = x^3+y^2-6xy$

$f_x = 3x^2-6y \quad f_y = 2y-6x \quad f_{xx} = 6x \quad f_{yy} = 2$

$f_{xy} = -6$

$3x^2-6y=0 \qquad 2y-6x=0$

$3x^2=6y \qquad 2y=6x$

$x^2=2y \qquad y=3x$

$y=\frac{x^2}{2}$

$\frac{x^2}{2}=3x$

$x^2=6x$

$x=6$

critical point is $(6,18)$ $(0,0)$

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$$D = f_{xx}f_{yy} - (f_{xy})^2$$
$$= (6\cdot 6)\cdot 2 - 36$$
$$= 36 > 0$$

$\therefore$ local minimum is 36 at $f(6, 18)$

at $(0,0)$ $\quad D = 0\cdot 2 - 36$
$$= -36 < 0$$

$\therefore$ saddle point is at $f(0,0)$

Q17. div $= 1+1+1 = 3$

$x = -9..11 \quad y = -12..8 \quad z = -6..14$

$$\int_{-6}^{14}\int_{-12}^{8}\int_{-9}^{11} 3 \; dx\, dy\, dz$$
$$= \int_{-6}^{14}\int_{-12}^{8} 60 \; dy\, dz$$
$$= \int_{-6}^{14} 1200 \; dz$$
$$= 24000$$