Yash Khangura Final Review Section 24 ask223@rutgers.edu

1. 1600
2. $z = 5x + 5y - 5$
3. abs max and abs min $= 0$
4. -2
5. -1
6. $z = 1 + y$
7. $\langle 1, 0, e^{3\pi/4} \rangle$
8. $21/2$
9. 0
10. ?
11. 162753.7914
12. 8
13. -200π NONSENSE!
14. 3
15. 3
16. $(0,0)$ and $(-9,0)$ are saddle points
17. 24000

1) Compute the line integral where C is the circle $x^2 + y^2 = 100$ traveled in the clockwise direction.

$$\int_{-10}^{10} \int_{-10}^{10} 7y\,dx + 3x\,dy = \int_{-10}^{10} \int_{-10}^{10} 4 \, dx\,dy = 1600$$

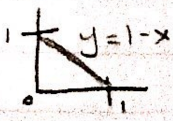
2) Find an equation of the tangent plane to the surface at the point $(1,1,5)$

$z = x^2 + 3xy + y^2$ $\qquad z = 1^2 + 3 \cdot 1 \cdot 1 + 1^2 = 5$

$f_x = 2x + 3y \rightarrow f_x = 5$ $\qquad z - 5 = 5(x-1) + 5(y-1)$

$f_y = 3x + 2y \rightarrow f_y = 5$ $\qquad z = 5x + 5y - 5$

3) Find the absolute maximum value and the absolute minimum value of the function $f(x,y) = x^2 y$ in the region $\{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$



$f_x = 2xy$ $\qquad f_y = x^2$ $\qquad f(0,0) = 0$

$0 = 2xy$ $\qquad 0 = x^2$ $\qquad$ abs max $= 0$

$y = 0$ $\qquad x = 0$ $\qquad$ abs min $= 0$

4) Compute $f_{xxyz}(0,0,0)$ if $f(x,y,z) = \sin(x^2 + y + z)$

$f_x = 2x\cos(x^2 + y + z)$

$f_{xx} = 2\cos(x^2 + y + z) - 4x^2 \sin(x^2 + y + z)$

$f_{xxy} = -2\sin(x^2 + y + z) - 4x^2 \cos(x^2 + y + z)$

$f_{xxyz} = -2\cos(x^2 + y + z) + 4x^2 \sin(x^2 + y + z)$

$= -2 \cdot \cos(0) = -2$

5.) Find dz/dy at the point $(1,1,1)$ if (x,y,z) are related by

$xy + xz + yz + x^2y^2z^2 = 4$

$x + x\,dz/dy + z + y\,dz/dy + 2x^2yz^2 + 2x^2y^2z\,dz/dy = 0$

$\frac{dz}{dy} = \frac{-x-z-2x^2yz^2}{x+y+2x^2y^2z} = \frac{-1-1-2}{1+1+2} = -\frac{4}{4} = -1$

6.) Find an equation for the plane that contains bothe the line $x = 1+t,\ y = 2+t,\ z = 3+t\ (-\infty < t < \infty)$ and the line $x = -t,\ y = 1+t,\ z = 2+t\ (-\infty < t < \infty)$

$\langle 1,1,1\rangle \times \langle -1,1,1\rangle = \langle 0,-2,2\rangle \quad -2y + 2z = 0 \quad (0,1,2)$

$1+t = -s \quad 2+t = 1+s \quad 3+t = 2+s$

$-t = 1+s \quad t = -1 \quad s = 0$

$0(x-0) - 2(y-1) + 2(z-2) = 0$

$-2y + 2 + 2z - 4 = 0$

$2z = 2 + 2y \rightarrow z = 1+y$

7.) A certain particle has acceleration given by $a(t) = \langle -4\sin 2t, -4\cos 2t, 9e^{3t}\rangle$. If its velocity at $t=0$ is $\langle 2,0,3\rangle$ and its position at $t=0$ is $\langle 0,1,1\rangle$, finds its position at the time $t = \pi/4$.

$v(t) = \langle 2\cos 2t, -2\sin 2t, 3e^{3t}\rangle$

$v(0) = \langle 2, 0, 3\rangle$ ✓

$r(t) = \langle \sin 2t, \cos 2t, e^{3t}\rangle \quad r(0) = \langle 0,1,1\rangle$ ✓

$r(\pi/4) = \langle 1, 0, e^{3\pi/4}\rangle$

8.) Compute the (scalar-function) line-integral $\int_C (x+y+2z)\,ds$ where the curve C is given by the parametric equation

$r(t) = \langle t, 2t, 2t\rangle \quad 0 \le t \le 1$

$\int_0^1 (t + 2t + 2\cdot 2t)\cdot\sqrt{(1)^2+(2)^2+(2)^2}\,dt = \int_0^1 7t\cdot 3\,dt = \frac{21t^2}{2} = \frac{21}{2}$

9.) if $\lim_{(x,y,z)\to(1,1,1)} f(x,y,z) = 1$, $\lim_{(x,y,z)\to(1,1,1)} g(x,y,z) = 2$ compute

$\lim_{(x,y,z)\to(1,1,1)} \sin\left(\frac{\pi}{3} f(x,y,z)\right)\cos\left(\frac{\pi}{4} g(x,y,z)\right) = 0$

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10) Compute $\iint_S F \cdot ds$, where $F = \langle x^2 + \sin(y+z), y^2 + xz^3, z^2 + e^{xy} \rangle$ and where S is the boundary (consisting of all six faces) of the cube $\{(x,y,z) \mid 0 \le x,y,z \le 1\}$ with the normal pointing outward

11) By finding a function f such that $F = \nabla f$, evaluate $\int_C F \cdot dr$ along the given curve C.

$F(x,y,z) = \langle 2e^{2x+3y+4z}, 3e^{2x+3y+4z}, 4e^{2x+3y+4z} \rangle$ $\quad C: x = t, y = 2t, z = t^2$, $0 \le t \le 1$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 2e^{2x+3y+4z} & 3e^{2x+3y+4z} & 4e^{2x+3y+4z} \end{vmatrix} = \langle 0, 0, 0 \rangle$$

$r(1) = \langle 1, 2, 1 \rangle$
$r(0) = \langle 0, 0, 0 \rangle$

F is conservative. Therefore $f = e^{2x+3y+4z}$

$$\int_0^1 \langle 2e^{8t+4t^2}, 3e^{8t+4t^2}, 4e^{8t+4t^2} \rangle \cdot \langle 1, 2, 2t \rangle \, dt$$

$$\int_0^1 8e^{8t+4t^2} + 8te^{8t+4t^2} dt = \int_0^1 8e^{8t+4t^2}(1+t)\, dt$$

$= 162753.7914$

12) Evaluate the line integral $\int_C 5y\,dx + 5x\,dy + 6z\,dz$ where $C: x = t^2, y = t, z = t^2, \quad 0 \le t \le 1$

$$\int_0^1 5t \cdot 2t + 5t^2 \cdot 1 + 6t^2 \cdot 2t \; dt = \int_0^1 10t^2 + 5t^2 + 12t^3 \, dt$$

$$= \int_0^1 15t^2 + 12t^3 \, dt = 5t^3 + 3t^4 \Big|_0^1 = 8$$

13) Evaluate $\iiint_E \frac{1}{\sqrt{x^2+y^2+z^2}} \, dV$ where E is the hemisphere $\{(x,y,z) \mid x^2+y^2+z^2 \le 100, z \ge 0\}$

?

$$\int_0^{2\pi}\int_{\pi}^{2\pi}\int_0^{10} \frac{1}{r} \cdot r^2 \sin\phi \; dr\, d\phi\, d\theta = \int_0^{2\pi}\int_{\pi}^{2\pi}\int_0^{10} r\sin\phi \; dr\, d\phi\, d\theta = \int_0^{2\pi}\int_{\pi}^{2\pi} \frac{r^2}{2} \sin\phi \Big|_0^{10} d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_{\pi}^{2\pi} 50\sin\phi \; d\phi\, d\theta = \int_0^{2\pi} -50\cos\phi \Big|_{\pi}^{2\pi} d\theta = \int_0^{2\pi} -100 \; d\theta = -100\theta \Big|_0^{2\pi} = -200\pi$$

NONSENSE!

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14) Evaluate the quadruple integral $\iiiint_E 360x\,dV$ where $E = \{(x,y,z,w) \mid 0 \le w \le 1,\ 0 \le z \le w,\ 0 \le y \le z,\ 0 \le x \le y\}$

$$\int_0^1\int_0^w\int_0^z\int_0^y 360x\,dx\,dy\,dz\,dw = \int_0^1\int_0^w\int_0^z 180y^2\,dy\,dz\,dw =$$

$$\int_0^1\int_0^w 60z^3\,dz\,dw = \int_0^1 15w^4\,dw = 3w^5\Big|_0^1 = 3$$

15) Find the Jacobian of the transformation from (u,v)-space to (x,y)-space $x = 3\sin(2u+v)$, $y = u+v+\cos(u+v)$ at the point $(u,v) = (0,0)$

$$J = \begin{vmatrix} dx/du & dx/dv \\ dy/du & dy/dv \end{vmatrix} = \begin{vmatrix} 6\cos(2u+v) & 3\cos(2u+v) \\ 1-\sin(u+v) & 1-\sin(u+v) \end{vmatrix} = \begin{vmatrix} 6 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 6 - 3 = 3$$

16) Find the local maximum and minimum points and saddle point(s) of the function $f(x,y) = x^3 + y^2 - 6xy$

$$x^3 + y^2 - 6xy = 0$$
$$y^2 - 6xy + 9x^2 = -x^3$$
$$(y-3x)^2 = -x^3$$
$$y - 3x = \sqrt{-x^3}$$
$$y = \sqrt{-x^3} + 3x$$

Critical points: $(0,0)$ $(-9,0)$

$$f_x = 3x^2 - 6y$$
$$f_y = 2y - 6x$$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2$$

$(0,0)$ $$= \begin{vmatrix} 6x & -6 \\ -6 & 2 \end{vmatrix} = 12x - 36 = -36$$

$(-9,0)$ $$= \begin{vmatrix} 6x & -6 \\ -6 & 2 \end{vmatrix} = 12x - 36$$
$$= -108 - 36 = -144$$

Both the critical points $(0,0)$ and $(-9,0)$ are saddle points b/c $D<0$ for both according to the Second Derivative Test

17) Use the Divergence Theorem to calculate the surface integral $\iint_S F \cdot ds$, where $F(x,y,z) = \langle x+y,\ y+z,\ x+z \rangle$, where S is the sphere (center $(1,-2,4)$ and radius 10), in other words the region in 3D space $\{(x,y,z) \mid (x-1)^2 + (y+2)^2 + (z-4)^2 = 100\}$

$$\operatorname{div} F = \tfrac{d}{dx}(x+y) + \tfrac{d}{dy}(y+z) + \tfrac{d}{dz}(x+z) = 1+1+1 = 3$$

$$\int_{-6}^{14}\int_{-12}^{8}\int_{-9}^{11} 3 = 24000$$