

1. $p=7y$ $Q=3x$

$$\frac{dp}{dx} - \frac{dQ}{dy} = 3 - 7 = -4$$

$-4 \times \pi \times 100 = -400\pi$
 because is clockwise so
 it is in the negative direction.

So, $-400\pi \times (-1) = 400\pi$

2. $z = x^2 + 3xy + y^2$

$$\frac{dz}{dx} = 2x + 3y = 2 \times 1 + 3 \times 1 = 5$$

$$\frac{dz}{dy} = 3x + 2y = 3 \times 1 + 2 \times 1 = 5$$

$$z - 5 = 5(x-1) + 5(y-1)$$

$$z = 5x - 5 + 5y - 5 + 5$$

$$z = 5x + 5y - 5$$

3. $f(x,y) = x^2y$ $(0,0,0)$

$$f_x = 2xy$$

$$f_y = x^2$$

$$f_{xx} = 2y$$

$$f_{yy} = 2x$$

$$f_{xy} = 2x$$

$$x=0 \quad 0 \leq y \leq 1-x$$

absolute minimum
 and absolute maximum

$$x^2y = 0 \quad x^2 = 0 \text{ is } 0.$$

$$x=0 \quad y=0$$

$$0 \leq x \leq 1 \quad 0 \leq y \leq 1-x$$

$$f_{xx} = 0 \quad f_{yy} = 0 \quad f_{xy} \neq 0$$

4. $\frac{d^2r}{dx^2 dy dz}$

$$= \frac{d^2}{dx^2} \cdot \frac{d}{dy} \cdot \frac{d}{dz}$$

$$= \sin$$

$$(-\sin(x^2 + y + z)) \sqrt{x^2 + 2\cos(x^2 + y + z)}$$

$$\cos(x^2 + y + z) \cdot \cos(x^2 + y + z)$$

5.

$$xy + xz + yz + x^2y^2z^2 = 4$$

$$x + xz' + z + yz' + x^2(2yz^2 + 2zy^2z') = 0$$

$$xz' + yz' + 2z x^2 y^2 z' = -x - z - 2x^2 y z^2$$

$$z'(x + y + 2z x^2 y^2) = -x - z - 2x^2 y z^2$$

$$z' = \frac{-x - z - 2x^2 y z^2}{x + y + 2z x^2 y^2}$$

plug in $(1,1,1)$

$$z' = \frac{-1-1-2}{1+1+2} = \frac{-4}{4} = -1$$

6

~~$$x-1=t \quad y-2=t \quad z-3=t$$~~

~~$$-x=t \quad y-1=t \quad z-2=t$$~~

$$L_1 = (1, 2, 3) + t(1, 1, 1)$$

$$L_2 = (0, 1, 2) + t(-1, 1, 1)$$

$$t=0 : (1, 2, 3), (0, 1, 2)$$

i	j	k
1	1	1
-1	1	1

$$0 - j(2) + k = (0, -2, 2)$$

$$0(x-0) - 2(y-2) + 2(z-3) = 0$$

$$-2y + 4 + 2z - 6 = 0$$



$$7. \text{a)} = (-4 \sin t, -4 \cos t, 9e^{3t})$$

$$\text{v)} = (2 \cos t, -2 \sin t, 3e^{3t})$$

$$t=0 \quad v(0) = (2, -2, 3)$$

$c=0$

$$\text{u)} = (2 \cos t, -2 \sin t, 3e^{3t})$$

$$\text{p)} = (\sin t, \cos t, e^{3t})$$

$$t=0 \quad v(0) = (1, 1, 3)$$

$c=0$

$$\text{p)} = (\sin t, \cos t, e^{3t}) \quad t = \frac{\pi}{4}$$

$$= (1, 0, e^{\frac{3\pi}{4}})$$

$$8. \text{a)} = (t, t, t)$$

$$x=t \quad y=t \quad z=t$$

$$\int_0^1 \sqrt{t^2 + t^2 + t^2} dt$$

$$\equiv x'(t) = 1$$

$$y'(t) = 2 \quad z'(t) = 2$$

$$\sqrt{1+4+4} = 3$$

$$\int_C x + y + z \, ds$$

$$= \int_0^1 (t + t + t) \cdot 3 \, dt$$

$$= \int_0^1 3t^2 \, dt$$

$$= \frac{3}{2}$$

9.

$$\lim_{(x,y,z) \rightarrow (1,1,1)} \sin\left(\frac{\pi}{3}x\right) \cos\left(\frac{\pi}{4}yz\right)$$

$$= \lim_{(x,y,z) \rightarrow (1,1,1)} \sin\frac{\pi}{3} \cos\frac{\pi}{2}$$

$$= 0$$

10.

$$P = x^2 + \sin y + z$$

$$Q = y^2 + xz$$

$$R = z^2 + e^{xy}$$

$$z=1$$

$$11. \int_C ze^{2x+3y+4z} dx + ze^{2x+3y+4z} dy + ze^{2x+3y+4z} dz$$

$$x=t \quad y=2t \quad z=t^2 \quad 0 \leq t \leq 1$$

$$\int_0^1 ze^{2t+6t+4t^2} dt + \int_0^1 ze^{2t+6t+4t^2} dt + \int_0^1 2te^{2t+6t+4t^2} dt$$

$$= \int_0^1 8te^{2t+6t+4t^2} + 2te^{2t+6t+4t^2} dt$$

$$= 162753.79$$



12.
 $x=t^2 \quad y=2t \quad z=t^2$
 $0 \leq t \leq 1$

$$\int_0^1 \int_0^1 \int_0^1 st \cdot z \, dt \, ds \, dz + \int_0^1 \int_0^1 st^2 \, ds \, dz + \int_0^1 \int_0^1 6t^2 \cdot zt \, ds \, dz$$

$$= \int_0^1 \int_0^1 10t^2 + 5t^2 + 12t^3 \, ds \, dz$$

$$= 8$$

13.

$$x^2 + y^2 + z^2 \leq 100 \quad z \geq 0$$

$$z = \sqrt{100 - x^2 - y^2}$$

$$\int \int \int \sqrt{(\cos \theta)^2 + (\sin \theta)^2 + z^2} \, dV$$

$$0 \leq r \leq 10 \quad 0 \leq \theta \leq 2\pi$$

16.

$$f_x = 3x^2 - 6y$$

$$f_y = 2y - 6x$$

$$f_{xx} = 6x^2$$

$$f_{yy} = -6$$

$$f_{xy} = -2$$

$$3x^2 - 6y = 0 \quad 2y - 6x = 0$$

$$(x, y) = (6, 18)$$

$$(x, y, z) = (6, 18, 0)$$

$$f_{xx} f_{yy} - (f_{xy})^2 = 216 \cdot 2 - 36 = 396$$

$D > 0$ $f_{xx} > 0$ so

$(6, 18)$ is a local minimum

$$f_{xx} f_{yy} - (f_{xy})^2 = 0 \cdot 2 - 36$$

$$= -36$$

$D < 0$ is a saddle point.

14. $\int_0^1 \int_0^w \int_0^y \int_0^z 3xyz \, dx \, dy \, dz \, dw$

$$= 3$$

15. $\frac{dx}{du} \quad \frac{dx}{dv} = 6 \cos(2u+2v) \quad 3 \cos(2u+2v)$

$$\frac{dy}{du} \quad \frac{dy}{dv} = 1 - \sin(u+v) \quad 1 - \sin(u+v)$$

$$= 6 \cos(0) \quad 3 \cos(0)$$

$$= 1 - \sin(0) \quad 1 - \sin(0)$$

$$= \begin{matrix} 6 \\ 1 \end{matrix} \quad \begin{matrix} 3 \\ 1 \end{matrix} = 6 - 3 = 3$$



17.

$$\begin{aligned}\operatorname{div} F &= \frac{d}{dx} xfg + \frac{d}{dy} gfc + \frac{d}{dz} xtz \\ &= 1 + 1 + 1 = 3\end{aligned}$$

$$\begin{aligned}3 \times \pi V^2 &= 3 \times \pi \times 10^2 \\ &= 300\pi\end{aligned}$$

