Lecture 26 attendance quiz Shaun Goda

January February March April May June July August September October November December

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

1) $\int_C 7y\,dx + 3x\,dy = \int_0^{2\pi} -70\sin t(-10\sin(t))\,dt + \int_0^{2\pi} 30\cos t(-10\cos t)\,dt$

$x = r\cos(t) = 10\cos t$

$y = -r\sin(t) = -10\sin t$

$0 \leq t \leq 2\pi$

$= \int_0^{2\pi} (700\sin^2 t - 300\cos^2 t)\,dt$

$= \left. 200t - 500\sin t\cos t \right|_0^{2\pi} = \boxed{400\pi}$

2) $f(x,y) = x^2 + 3xy + y^2$ $\quad f(1,1) = 1^2 + 3 + 1^2 = 5$

$f_x = 2x + 3y$ $\quad f_x(1,1) = 2 + 3 = 5$

$f_y = 3x + 2y$ $\quad f_y(1,1) = 3 + 2 = 5$

$z - 5 = 5(x-1) + 5(y-1)$

$z - 5 = 5x - 5 + 5y - 5$

$\boxed{-15 = 5x + 5y - z}$

3) $f_x = 2xy \quad f_y = x^2$

$2xy = 0 \quad x^2 = 0$

$2y = x$

$y = \frac{x}{2}$

$x^2 = 0 \quad f(0,0) = 0$

$x = 0$

$y = \frac{0}{2}$

$y = 0$

$0.5^2\, 0.5$

$\boxed{\text{absolute min : } 0}$

$\boxed{\text{absolute max : } 0.125}$

4) $f(x,y,z) = \sin(x^2+y+z)$

$f_x = 2x\,\cancel{\sin}\cos(x^2+y+z)$

$f_{xx} = -4x^2\sin(x^2+y+z) + 2\cos(x^2+y+z)$

$f_{xxy} = -4x^2\cos(x^2+y+z) - 2\sin(x^2+y+z)$

$f_{xxyz} = 4x^2\sin(x^2+y+z) - 2\cos(x^2+y+z)$

$f_{xxyz}(0,0,0,0) = \boxed{-2}$

5) $xy + xz + yz + x^2y^2z^2 = 4$

when $a = x^2y^2$, $b = x+y$, $c = xy-4$

$z = \frac{-(x+y) \pm \sqrt{(x+y)^2 - 4(x^2y^2)(xy-4)}}{2x^2y^2}$

$\frac{\partial z}{\partial y}(1,1,1) = \boxed{-1}$

6) $\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = (1-1)i - (1+1)j + (1+1)k$
$= \langle 0, -2, 2 \rangle$

$0(x-1) - 2(y-2) + 2(z-3) = 0$
$-2y+4 + 2z - 6 = 0$
$\boxed{-2y + 2z = 2}$

7) $a(t) = \langle -4\sin 2t, -4\cos 2t, 9e^{3t} \rangle$

$v(t) = \langle 2\cos(2t) + c, -2\sin(2t) + c, 3e^{3x} + c \rangle$
$v(0) = \langle 2 + c = 2, c = 0, 3 + c = 3 \rangle$
$c = 0, \quad c = 0, \quad c = 0$
$p(t) = \langle \sin(2t) + c, \cos(2t) + c, e^{3t} + c \rangle$
$p(0) = \langle c = 0, 1 + c = 1, 1 + c = 1 \rangle$
$p(\frac{\pi}{4}) = \langle \sin(\frac{\pi}{2}), \cos(\frac{\pi}{2}), e^{\frac{3\pi}{4}} \rangle$
$= \boxed{\langle 1, 0, e^{\frac{3\pi}{4}} \rangle}$

8) $\int_C (x+y+2z)\,ds = \int_0^1 (t+2t+4t)\sqrt{(1)^2+(2)^2+(2)^2}\,dt$
$= 3\int_0^1 (t+2t+4t)\,dt = 3\left|\frac{t^2}{2} + t^2 + 2t^2\right|_0^1$
$= 3(\frac{1}{2} + 1 + 2) = \boxed{10.5}$

9) $\sin(\frac{\pi}{3}(1))\cos(\frac{\pi}{4}(2)) = \boxed{0}$

10) $\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\,dS = \iiint_V \langle x^2 + \sin(y+z), y^2 + xz^3, z^2 + e^{xy} \rangle \cdot n$

$\int_0^1 \int_0^1 \int_0^1 (2x + 2y + 2z)\,dx\,dy\,dz$

$\int_0^1 (2x + 2y + 2z)\,dx = \left| x^2 + 2yx + 2zx \right|_0^1 = 1 + 2y + 2z$
$\int_0^1 (1 + 2y + 2z)\,dy = \left| y + y^2 + 2zy \right|_0^1 = 1 + 1 + 2z$
$\int_0^1 (2 + 2z)\,dz = \left| 2z + z^2 \right|_0^1 = 2 + 1 = \boxed{3}$

11) $f_x = 2e^{2x+3y+4z}$

$f = e^{2x+3y+4z} + g(y,z)$

$f_y = 3e^{2x+3y+4z}$ $\qquad g(y,z) = 0$

$f = e^{2x+3y+4z} \quad + g(z)$

$f_z = 4e^{2x+3y+4z}$

$f = e^{2x+3y+4z} \quad + 0$

$f = e^{2x+3y+4z}$

$r(t) = \langle t, 2t, t^2 \rangle \quad 0 \le t \le 1$

$r(0) = \langle 0, 0, 0 \rangle$

$r(1) = \langle 1, 2, 1 \rangle$

$f(1,2,1) - f(0,0,0)$

$= e^{2+6+4} - e^0 = \boxed{e^{12} - 1}$

12) $\int_C 5y\,dx + 5x\,dy + 6z\,dz$

$\int_0^1 5t(2t)\,dt = \left| \frac{10t^3}{3} \right|_0^1 = \frac{10}{3}$

$\int_0^1 5t^2(1)\,dt = \left| \frac{5t^3}{3} \right|_0^1 = \frac{5}{3}$

$\int_0^1 6t^2(2t)\,dt = \left| 3t^4 \right|_0^1 = 3$

$\frac{10}{3} + \frac{5}{3} + 3 = \boxed{8}$

13) $\iiint_E \frac{1}{\sqrt{x^2+y^2+z^2}}\,dV = \int_{\frac{\pi}{2}}^{\pi} \int_0^{2\pi} \int_0^{10} \frac{\rho^2 \sin\varphi}{\sqrt{\rho}}\,d\rho\,d\theta\,d\varphi$

$\int_0^{10} \frac{\rho^2 \sin\varphi}{\sqrt{\rho}}\,d\rho = \left| \frac{2\rho^{\frac{5}{2}}}{5} \right|_0^{10} = 40\sqrt{10} \cdot \sin\varphi$

$40\sqrt{10} \int_0^{2\pi} \sin\varphi\,d\theta = 80\pi\sqrt{10} \sin\varphi$

$\int_{\frac{\pi}{2}}^{\pi} 80\pi\sqrt{10} \sin\varphi\,d\varphi = 80\pi\sqrt{10} \left| -\cos\varphi \right|_{\frac{\pi}{2}}^{\pi} = \boxed{80\pi\sqrt{10}}$

14) $\iiiint_E 360x \, dV = \int_0^1 \int_0^w \int_0^z \int_0^y 360x \, dx\,dy\,dz\,dw$

$\int_0^y 360x \, dx = \left| 180x^2 \right|_0^y = 180y^2$

$\int_0^z 180y^2 \, dy = \left| 60y^3 \right|_0^z = 60z^3$

$\int_0^w 60z^3 \, dz = \left| 15z^4 \right|_0^w = 15w^4$

$\int_0^1 15w^4 \, dw = \left| 3w^5 \right|_0^1 = \boxed{3}$

15) $x_u = 6\cos(2u+v)$ $\qquad y_u = 1 - \sin(u+v)$
$x_v = 3\cos(2u+v)$ $\qquad y_v = 1 - \sin(u+v)$

$$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 6\cos(2u+v) & 3\cos(2u+v) \\ 1-\sin(u+v) & 1-\sin(u+v) \end{vmatrix}$$

$= (6\cos(2u+v) - 6\cos(2u+v)\sin(u+v))$
$- (3\cos(2u+v) - 3\cos(2u+v)\sin(u+v))$

@ $(u,v) = (0,0) \Rightarrow (6 - 6\cdot 0) - (3 - 3\cdot 0) = \boxed{3}$

16) $f(x,y) = x^3 + y^2 - 6xy$
$f_x = 3x^2 - 6y \quad f_{xx} = 6x \qquad f_{xy} = -6$
$f_y = 2y - 6x \quad f_{yy} = 2$

$3x^2 - 6y = 0 \qquad 2y - 6x = 0$
$3x^2 - 18x = 0 \leftarrow 2y = 6x$
$3x(x-6) = 0$
$x = 0,\ x = 6$
$y = 0,\ y = 18$ $\quad \Rightarrow (0,0),\ (6,18)$

$f_{xx}(0,0) = 0 \qquad f_{xx}(6,18) = 36$
$f_{xy}(0,0) = -6 \qquad f_{xy}(6,18) = -6$
$f_{yy}(0,0) = 2 \qquad f_{yy}(6,18) = 2$

$D = (0)(2) - (-6)^2 = -36 \qquad D = (36)(2) - (-6)^2 = 36$

Saddle point at (0,0) $\qquad$ local minimum at (6,18)

17) $\iint_S \langle x+y, y+z, x+z \rangle \, dS = \iiint_E (1+1+1)\, dV$

$\iiint_E 3\, dV = \int_0^{\pi}\int_0^{2\pi}\int_0^{10} 3\rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$

$\int_0^{10} 3\rho^2 \sin\varphi \, d\rho = \left| \rho^3 \sin\varphi \right|_0^{10} = 1000\sin\varphi$

$\int_0^{2\pi} 1000\sin\varphi \, d\theta = \left| 1000 \sin\varphi \, \theta \right|_0^{2\pi} = 2000\pi \sin\varphi$

$\int_0^{\pi} 2000\pi \sin\varphi \, d\varphi = \left| -2000\pi \cos\varphi \right|_0^{\pi} = \boxed{4000\pi}$