2017 Final

1.) $\int_C 7y\,dx + 3x\,dy$, $C = x^2+y^2=100$ cw direction

$P = 7y \quad Q = 3x, \quad P_y = 7 \quad Q_x = 3$

$\iint_D (3-7)\,dx\,dy \rightarrow \int_0^{2\pi}\int_0^{10} -4r\,dr\,d\theta =$

$\int_0^{10} -4r\,dr = -2r^2\Big|_0^{10} = -200 \qquad \int_0^{2\pi} -200\,d\theta = -200\theta\Big|_0^{2\pi} = -400\pi(-1) = \boxed{400\pi}$

2.) $z = x^2+3xy+y^2$ tangent plane @ $(1,1,5)$

$f(x,y,z) = x^2+3xy+y^2-z$

$\nabla f = \langle 2x+3y,\ 3x+2y,\ -1\rangle$

$\nabla f(1,1,5) = (2(1)+3(1),\ 3(1)+2(1),\ -1)$

$= (5,\ 5,\ -1)$

$0 = (5,5,1)\cdot(x-5,\ y-5,\ x+1) = \boxed{5x+5y+z=49}$ or $\boxed{z = 49-5x-5y}$

3.) $f(x,y) = x^2y \quad \{(x,y): 0 \le x \le 1,\ 0 \le y \le 1-x\}$

$f_x = 2xy, \quad f_y = x^2 \qquad x = \frac{2}{3}$

$\boxed{\text{min value} = 0}$

$x = \frac{2}{3} \qquad y = \frac{1}{3}$

$f(\frac{2}{3}, \frac{1}{3}) = (\frac{2}{3})^2(1-\frac{2}{3}) = (\frac{2}{3})^2(\frac{1}{3}) = 0.\overline{148}$

$\boxed{\text{max value} = 0.\overline{148}}$

4.) f_{xxyz} $(0,0,0)$ of $f(x,y,z) = \sin(x^2+y+z)$

$f_x = 2x\cos(x^2+y+z) \quad f_{xx} = 2\cos(x^2+y+z) - 4x^2\sin(x^2+y+z)$

$f_{xxy} = -2\sin(x^2+y+z) - 4x^2\cos(x^2+y+z) \qquad f_{xxyz} = 4x^2\sin(x^2+y+z) - 2\cos(x^2+y+z)$

$0 - 2(1) = \boxed{-2}$

5.) $\frac{\partial z}{\partial y}$ @ $(1,1,1)$ $\quad xy+xz+yz+x^2y^2z^2 = 4$

$f_y = x+z+2yx^2z^2 = 0 \rightarrow$

$x + zz' + 2x^2zz'(yz)' \rightarrow$ ~~$x+zz'+2x^2zz'(y'z+yz')=0$~~

$x + (zz')(2x^2(yz)') = 0$

$zz'(2x^2(y'z+yz')) = -x \rightarrow zz'(2x^2y'z + 2x^2yz') = -x$

$2x^2y'z^2z' + 2x^2yzz'^2 = -x \rightarrow 2x^2y'z^2 + 2x^2yzz' = \frac{-x}{z'}$

$2(1)(1)(1) + 2(1)(1)(1)z' = \underline{-(1)}$

$\boxed{0}$

6.) $x=1+t$, $y=2+t$, $z=3+t$

$x=-t$, $y=1+t$, $z=2+t$

$\langle 1,1,1\rangle$ } $\rightarrow \langle 1,1,1\rangle \times \langle -1,1,1\rangle = \langle 0,-2,2\rangle$

$\langle -1,1,1\rangle$

$0(x-1) + -2(y-2) + 2(z-3) = 0$

$\boxed{-2y+2z=2}$ or $\boxed{z=1+y}$

7.) $a(t)=\langle -4\sin 2t, -4\cos 2t, 9e^{3t}\rangle$

@ $t=0$ $v=\langle 2,0,3\rangle$ $p=\langle 0,1,1\rangle$

$v(t)=\langle 2\cos 2t, -2\sin 2t, 3e^{3t}\rangle$

$p(t)=\langle \sin 2t, \cos 2t, e^{3t}\rangle$

$p(\frac{\pi}{4}) = (\sin 2(\frac{\pi}{4}), \cos 2(\frac{\pi}{4}), e^{3(\frac{\pi}{4})})$

$\boxed{= (\sin\frac{\pi}{2}, \cos\frac{\pi}{4}, e^{\frac{3\pi}{4}})}$

8.) $\int_c (x+y+2z)\,ds$

$r(t)=\langle t, 2t, 2t\rangle$, $0 \le t \le 1$

$\langle t, 2t, 4t\rangle$

mag $= \sqrt{1^2+2^2+4^2} = \sqrt{21}$

~~$\int_0^1 (t+2t+4t)\sqrt{21}\,dt$~~ $\int_0^1 t+2t+4t\,(\sqrt{21})\,dt$

$\int_0^1 7t\sqrt{21}\,dt \rightarrow \boxed{\frac{7\sqrt{21}}{2}}$

9.) $\lim_{(1,1,1)} \sin\frac{\pi}{3}(1)\cos\frac{\pi}{4}(2)$

$\lim_{(1,1,1)} \sin\frac{\pi}{3}\cdot\cos\frac{\pi}{2}$

$\frac{\sqrt{3}}{2}\cdot 0 = \boxed{0}$

10.) $F=\langle x^2+\sin(y+z), y^2+xz^3, z^2+e^{xy}\rangle$

div $F = 2(x\sin(x+y)+y+z)$

$\int_0^1\int_0^1\int_0^1 2(x\sin(x+y)+y+z)\,dV \rightarrow \int_0^1 2(x\sin(x+y)+y+z)\,dx$

$\int_0^1 2(z+\sin(y+1)-\cos(y+1)-\sin(y)+y)\,dy$

$\int_0^1 2z-2\sin(2)-2\cos(2)+2\sin(1)+4\cos(1)-1 = -2(\sin(2)+\cos(2)-\sin(1)-2\cos(1)) =$

$\boxed{2.858}$

11.) $F = \nabla f$

$F = \langle 2e^{2x+3y+4z}, 3e^{2x+3y+4z}, 4e^{2x+3y+4z} \rangle$

$x = t$, $y = 2t$, $z = t^2$ $\quad 0 \le t \le 1$

$f = \langle e^{2x+3y+4z}, e^{2x+3y+4z}, e^{2x+3y+4z} \rangle$ → property of derivative of e

$\langle e^{2t+3(2t)+4t^2}$, " , " $\rangle$

$\int_0^1 F \cdot dr = \int_0$ [illegible] $= \int_0$ [illegible]

$(e^{2x+3y+4z})' = r'(t) = (8t+8)e^{4t^2+8t} \cdot r(t) =$

$3e^{8t^2+16t}(8t+8)$ $\quad \int_0^1 3e^{8t^2+16t}(8t+8) = \boxed{\frac{3e^{24}-3}{2}}$

actually yes for the answer. did not mean to cross out

12.) $\int_C 5y\,dx + 5x\,dy + 6z\,dz$

$5t \quad + 5(t^2) + \quad 6(t^2)$

$= \int_0^1 5(t)\overset{(t)}{}\,dt + 5(t^2)(2t)\,dt + 6t^2(2t)\,dt$

$= 25t + 10t^3 + 12t^3\,dt = \int_0^1 22t^3 + 21t\,dt = \frac{11t^4 + 25t^2}{2}\Big|_0^1 = \boxed{18}$

13.) $\iiint$ $\quad x = \rho\sin\varphi\cos\theta$

$y = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

$\Big\} \rightarrow \dfrac{1}{\sqrt{\rho^2}}$

$\int_0^{\pi}\int_0^{\pi}\int_0^{10} \frac{1}{\rho}\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi \rightarrow \int_0^{\pi}\int_0^{\pi}\int_0^{10} \rho\sin\varphi\,d\rho\,d\theta\,d\varphi$

$\int_0^{\pi} 50\sin\varphi \rightarrow 100 \quad \int_0^{\pi} \boxed{100\pi}$

14.) $\int_0^1 \int_0^w \int_0^z \int_0^y 360x \, dx\,dy\,dz\,dw$

$\int_0^z 180y^2 dy \to \int_0^w 60z^3 dz \to \int_0^1 15w^4 dw \to \boxed{3}$

15.) $x = 3\sin(2u+v)$ $\quad y = u+v+\cos(u+v)$

$$\begin{vmatrix} 6\cos(2u+v) & 3\cos(v+2u) \\ 1-\sin(u+v) & 1-\sin(u+v) \end{vmatrix} = 3\cos(2u+v) - 3\sin(u+v)\cos(2u+v)$$

@ $(0,0)$

$3\cos(0) - \sin(0)\cos(0)$

$3(1) - 0 = \boxed{3}$

16.) x^3+y^2-6xy

$d_x = 3x-6y$, $d_y = 2y-6x$, $f_{xx} = 3$, $f_{xy} = -6$, $f_{yy} = 2$

$3x-6y=0 \to 6x-12y=0 \quad 3x-6(0)=0$

$2y-6x=0 \quad -6x+2y=0 \quad 3x=0$

$-10y=0 \quad \boxed{x=0}$

$\boxed{y=0}$,

$D = 3(2) - (-6)^2 = 6+36 = 42$

$\boxed{(0,0) \text{ is a local min! b/c } D>0 \text{ + saddle pt } f_{xx}>0}$ no max,

17.) $F(x,y,z) = \langle x+y, y+z, x+z \rangle$

$\text{div}F = 3$

$r: 0 \text{ to } 10$

$\iiint 3 \, r\,dz\,dr\,d\theta \quad \int_0^{10} 3r\,dr \to \int_0^{2\pi} 150\,d\theta \quad \int_0^{2\pi} 300\pi = \boxed{600\pi z}$