12/12/20 Fall 2017 Final.

1) $\int_C 7y\,dx + 3x\,dy$

$C = x^2+y^2=100$, $r=10$

clockwise direction.

$x = 10\cos(t)$ $\quad dx = -10\sin t\,dt$.

$y = -10\sin(t)$ $\quad dy = -10\cos t\,dt$

$0 \le t \le 2\pi$.

$$\int_0^{2\pi} 7(-10\sin t)\cdot(-10\sin t)\,dt + \int_0^{2\pi} 3(10\cos t)(-10\cos t\,dt)$$

$$= \int_0^{2\pi} 700\sin^2 t\,dt + \int_0^{\pi} -300\cos^2 t\,dt$$

$$= 700\pi - 300\pi$$

$$= 400\pi$$

2) $z = x^2+3xy+y^2$. at $(1,1,5)$

$f_x = 2x+3y$

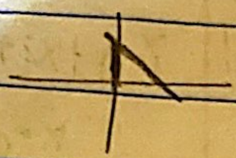
$f_y = 3x+2y$

$f_x(1,5) = \cancel{2+15=}7 \quad 2+3=5$

$f_y = 3\cancel{+2} = 3+2=5$

$L(x,y) = 5 - 5(x-1) + 5(y-1)$.

3) $\{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$



$f(x,y) = x^2 y$

$f_x = 2xy$ $\qquad x^2 = 0$

$f_y = x^2$ $\qquad x = 0$ $\quad$ y can be anything.

$0 \le y \le 1$

$f(0,0) = 0 \leftarrow$ critical point.

$f(0,1) = 0 \leftarrow$ Critical Point.

Absolute Minimum $\rightarrow (0,0)$

Absolute Max: $(0,1)$

4) $f(x,y,z) = \sin(x^2+y+z)$

$f_x = 2x\cos(x^2+y+z)$

$f_{xx} = 2\cos(x^2+y+z) + (-4x^2 \sin(x^2+y+z))$

$f_{xxy} = -2\sin(x^2+y+z) \quad -4x^2\cos(x^2+y+z)$

$f_{xxyz} = -2\cos(x^2+y+z) \quad +4x^2(x^2+y+z)$

$= -2 \qquad \nearrow 0$

5) $xy + xz + yz + x^2y^2z^2 = 4$

$\frac{dz}{dy} \mid x \cdot \frac{dz}{dy} + \left(x^2z^2 \cdot \frac{dz}{dy} \cdot 2y\right) = 0$

$\frac{dz}{dy}\left(x + x^2z^2 \cdot 2y\right) = 0$

$x^2y^2z^2$

5) $xy + xz + yz + x^2y^2z^2 = 4$

$x \cdot \frac{dz}{dy} + y \cdot \frac{dz}{dy}$

$1 + x\frac{dy}{dy} + \cancel{zd}\ y\frac{dy}{dy} + x^2y^2\frac{dz}{dy} + z^2 = 0$

$$\frac{-z^2 - 1}{(x^2y^2 + x + y)} = \frac{dz}{dy}$$

6) $x = 1 + t$ $\quad$ $x = -t$

$y = 2 + t$ $\quad$ $y = 1 + 1$

$z = 3 + t$ $\quad$ $z = 2 + t.$

$\langle 1, 2, 3 \rangle$ $\quad$ $\langle 0, 1, 2 \rangle$

$\langle 2, 1, 0 \rangle - \langle 1, 2, 3 \rangle$

$= \langle 1, -1, -3 \rangle$

$\langle 1, 1, 1 \rangle \times \langle 1, -1, -3 \rangle$

$= \langle 4, -2, -2 \rangle$

$(4, -2, -2) \cdot (1, 2, 3) = 4 - 4 - 6 = -6$

$4x - 2y - 2z = -6$

7). $a(t) = \langle -4\sin(2t), -4\cos(2t), 9e^{3t} \rangle$

$\int a(t)\,dt = \langle 2\cos 2t, -2\sin 2t, 3e^{3t} \rangle$ at $t=0$

$\langle 2, 0, 3 \rangle$

$\int v(t)\,dt = \langle \sin 2t, \cos 2t, e^{3t} \rangle$

at $t=0$, $x(t) = \langle 0, 1, 1 \rangle$

8) $\int_C (x+y+2z)\,ds$

C: $r(t) = \langle t, 2t, 2t \rangle$

$t: [0,1]$

$f(r(t)) = t + 2t + 4t = 7t.$

$r'(t) = \langle 1, 1, 2 \rangle$

$\|r'(t)\| = \sqrt{6}.$

$\int_0^1 7\sqrt{6}\,t\,dt = \frac{7\sqrt{6}}{2}t^2 = \frac{7\sqrt{6}}{2}.$

9) $\lim_{(x,y,z)\to(1,1,1)} f(x,y,z) = 1.$, $\lim_{(x,y,z\to 1,1,1)} g(x,y,z) = 2.$

$\lim_{(x,y,z)\to(1,1,1)} \sin\left(\frac{\pi}{3}(f(x,y,z))\cos\left(\frac{\pi}{4}, g(x,y,z)\right)\right)$

$\sin\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{4}\right)$

10) $\iint_S F \cdot dS$.

$F = \langle x^2 + \sin(y+z),\ y^2 + xz^3,\ z^2 + e^{xy} \rangle$

$\text{div}(F) = 2x + 2y + 2z$

$\int_0^1 \int_0^1 \int_0^1 \text{div}(F)\, dz\, dy\, dx = 3.$

11) $F(x,y,z) =$

$\langle 2e^{2x+3y+4z}$

$3e^{2x+3y+4z}$

$4e^{2x+3y+4z} \rangle$

$C: x = t,\ y = 2t,\ z = t^2$

$t: [0,1]$

$\int 2e^{2x+3y+4z}\, dx = e^{2x+3y+4z} + g(z)$

$\int 3e^{2x+3y+4z}\, dz = e^{2x+3y+4z}$

$\int 4e^{2x+3y+4z}\, dz = e^{2x+3y+4z}.$

$r(t) = \langle t,\ 2t,\ t^2 \rangle$

$r(1) - r(0)$

$(1,2,1) - (0,0,0) = e^{12} - e^0 = e^{12} - 1$

12) $\int 5y\,dy$ $\quad \int\int 5y\,dx + 5x\,dy + 6z\,dz.$

$$\int_0^1 (10t^2\ 15t^2 + 12t^3)\,dt$$

$$= \int_0^1 (15t^2 + 12t^3)\,dt = 8$$

13) $$\iiint_E \frac{1}{\sqrt{x^2+y^2+z^2}}\,dV$$

$$\{(x,y,z) \mid x^2+y^2+z^2 \le 100,\ z\le 0\}$$

$\frac{\pi}{2} \le \phi \le \pi \qquad 0 \le \theta \le 2\pi \qquad 0 \le \rho \le 10$

$$\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi}\int_0^{10} \sin\phi\,d\rho\,d\phi\,d\theta = 100\pi.$$

14) $$\iiiint_E 360\,x\,dV.$$

$E = \{(x,y,z,w) \mid 0 \le w \le 1, 0 \le z \le w, 0 \le y \le z, 0 \le x \le y\}.$

$$\int_0^1\int_0^w\int_0^z\int_0^y 360\,x\,dx\,dy\,dz\,dw.$$

$$\int_0^z 180y^2\,dy = 60z^3$$

$$\int_0^w 60z^3\,dz = 15w^4$$

$$\int_0^1 15w^4\,dw = 15$$

15) $x = 3\sin(2u+v)$ $(u,v)=(0,0)$

$y = u+v+\cos(u+v)$

$x_u = 6\cos(2u+v)$ $=6$

$x_v = 3\cos(2u+v)$ $=3$

$y_u = 1-\sin(u+v)$ $=1$

$y_v = 1-\sin(u+v)$ $=1$

$6-3=3$

16) $f(x,y)=x^3+y^2-6xy$

$f_x = 3x^2-6y$

$f_y = 2y-6x$ $2y-6x=0$

$f_{xx} = x$ $y=3x$

$f_{yy} = 4$ $3x^2-18x=0$

$f_{xy} = -6$ $x^2-6x=0$

$x(x-6)=0$

$(0,0)$: $D=-36$ $(0,0) \to$ Saddle $x=0$ $y=0$

$(6,18)$: $D=108$, $(6,18)$ is a local max. $x=6$ $y=18$.

17) $div(F) = 3$

$\iiint 3\,dV$

$\frac{4}{3}\pi R^3 \Rightarrow V = 4000\pi$.

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